

Radial and axial compression of a hollow electron beam using an asymmetric magnetic cusp

W. W. Destler and M. J. Rhee

University of Maryland, College Park, Maryland 20742

(Received 11 February 1977; final manuscript received 13 April 1977)

The injection of hollow, relativistic electron beams through a narrow asymmetric magnetic cusp field has been investigated analytically. Preliminary experimental results are in good agreement with the analytical theory.

The motion of charged particles in cusped magnetic fields has been studied extensively in recent years.¹⁻⁵ We wish to report analytical and experimental results which indicate that the injection of hollow, charged particle beams through asymmetric magnetic cusp fields may be a potentially simple and useful way of generating high local particle densities and self-colliding particle orbits. Possible applications of this technique include

studies of plasma heating and confinement, ion or electron beam pellet heating, colliding beam fusion, intense microwave generation, electron beam excited gas lasers, and ion sources.

Solutions for the motion of charged particles in an asymmetric magnetic cusp field may be found in the same manner as in the symmetric case.⁴ Figure 1 shows

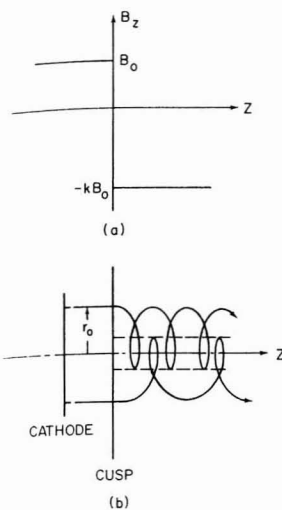


FIG. 1. (a) Ideal asymmetric magnetic cusp field. (b) Typical particle trajectories for two different electron energies.

schematically a fully idealized asymmetric cusp field configuration. A hollow electron beam is emitted from a circular cathode centered on the axis of symmetry of the cusp, and the beam is accelerated in the anode-cathode gap before the cusp transition. An infinitely small cusp transition is assumed, and the magnetic field as a function of axial position z may be written

$$B_z = B_0[1 - (1 + k)u(z)] \quad (1a)$$

and accordingly, the magnetic vector potential

$$A_\theta = (rB_0/2)[1 - (1 + k)u(z)], \quad (1b)$$

where u is the Heaviside unit step function and k is the ratio of the field on the downstream side of the cusp to the field on the upstream side.

A straightforward solution of the Lagrange equation for the z coordinate yields an expression for the axial velocity of the particle in the downstream region v_{z2} in terms of the initial axial velocity v_{z1} after acceleration in the diode, the cathode radius r_0 , and k :

$$v_{z2}^2 = v_{z1}^2 - \frac{1}{4}\omega_c^2 r_0^2 (k+1)^2, \quad (2)$$

where $\omega_c = eB_0/m\gamma$ is the relativistic electron cyclotron frequency. In the case $k=1$, the result for the symmetric cusp⁴ is obtained.

Similarly, the Lagrange equation for the r coordinate may be solved with the following result:

$$R_1^2 - \rho_1^2 = r_0^2, \quad z < 0, \quad (3a)$$

$$\rho_2^2 - R_2^2 = r_0^2/k, \quad z > 0, \quad (3b)$$

where ρ_1 and ρ_2 are the upstream and downstream Larmor radii and R_1 and R_2 are the distance from the axis of symmetry to the center of gyration of the particles upstream and downstream of the cusp transition. Again, the case $k=1$ recovers the symmetric cusp result. The central ray orbits may be obtained by noting that the total velocity of the electron is a constant of the motion; thus,

$$v_{z2}^2 + \rho_2^2 k^2 \omega_c^2 = v_{z1}^2 + \rho_1^2 \omega_c^2. \quad (4)$$

For the central ray, $\rho_1 = 0$ and

$$v_{z2}^2 + \rho_2^2 k^2 \omega_c^2 = v_{z1}^2. \quad (5)$$

Using Eq. (2), an expression is obtained for ρ_2 in terms of the cathode radius r_0 and k :

$$\rho_2 = [(k+1)/2k]r_0. \quad (6)$$

Typical particle orbits are shown in Fig. 2. Each central ray particle orbit is tangent to a circle about the axis of symmetry of radius given by

$$r_f = 2\rho_2 - r_0 = r_0/k \quad (7)$$

and to a circle drawn about r_0 . Thus, as k is increased, the particle density at $r=r_f$ should increase. As the field on the upstream side of the cusp is lowered to zero, all orbits will intersect the system axis.

A reasonable estimate of the beam density at $r=r_f$ (neglecting collective effects) may be obtained by noting that the total axial current must be a constant upstream and downstream of the cusp transition. Thus we may write

$$n_0 r_0 v_{z1} \approx n_f r_f v_{z2}, \quad (8)$$

where n_0 is the upstream beam density and n_f is the density at the inner focus $r=r_f$. In this approximation, the minor radial width of the beam at the inner focus is assumed to be produced by a finite Larmor radius ρ_1 upstream of the cusp and is therefore comparable to the upstream beam width to first order.⁴ Using Eqs. (2) and (7), an expression for the particle density at the inner focus is obtained:

$$\frac{n_f}{n_0} \approx k \left[1 - \left(\frac{\omega_c r_0}{v_{z1}} \right)^2 \left(\frac{k+1}{2} \right)^2 \right]^{-1/2}. \quad (9)$$

Thus, as k is increased, the particle density at the inner focus should increase dramatically. In addition, as the magnetic field is raised toward the cutoff value given by

$$1 - \left(\frac{\omega_c r_0}{v_{z1}} \right)^2 \left(\frac{k+1}{2} \right)^2 = 0, \quad (10)$$

the density should increase further. Thus, this technique produces both a radial and axial compression of the downstream beam. The actual final beam density at

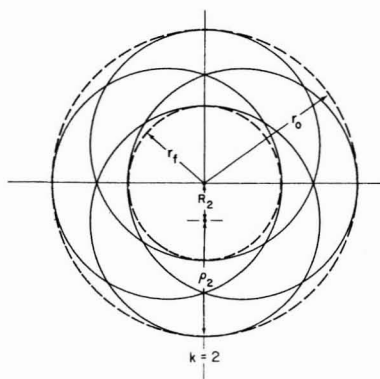


FIG. 2. Projection of downstream particle orbits onto a $z = \text{constant}$ plane for the case $k=2$.

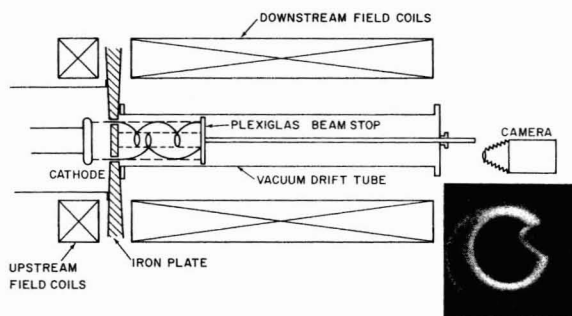


FIG. 3. General experimental configuration. Also shown is a typical photograph of the time-integrated downstream beam cross section for the case $k=2$. The distortion on the right side of the photograph is caused by the beam stop support rod.

the inner focus will be determined by the collective interactions of the beam electrons.

A simple experiment has been performed to check the analytical results using the facilities of the University of Maryland Electron Ring Accelerator Experiment.^{4,6,7} The general experimental configuration is shown in Fig. 3. In the experiment, typical diode voltage and current were 2.5 MV and 3–10 kA, respectively. The radius of the cathode was 6 cm. Photographs of the time-integrated downstream beam cross section were obtained by photographing the light produced by the beam when it strikes a Plexiglas beam stop located in the downstream drift region. The radius r_f of the inner focus of the beam was measured as a function of k and results are plotted in Fig. 4. It is easily seen that the agreement between the analytical model and the experimental results is very good. Although the experimental geometry did not allow photographing the beam cross section for the case where $r_f=0$, there is, in principle, no reason

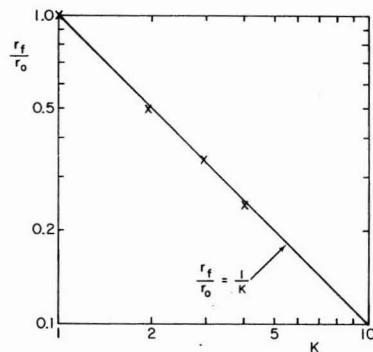


FIG. 4. Plot of the radius of the inner focus (normalized to r_0) as a function of k .

that the upstream magnetic field cannot be reduced to zero.

In conclusion, it appears that the injection of hollow, charged particle beams through asymmetric magnetic cusp fields is an attractive method of generating high local particle densities and self-colliding particle orbits.

This work was supported in part by the National Science Foundation under Grant No. PHY7611178.

¹G. Schmidt, Phys. Fluids **5**, 994 (1962).

²J. Sinnis and G. Schmidt, Phys. Fluids **6**, 841 (1963).

³M. Friedman, Phys. Rev. Lett. **24**, 1098 (1970).

⁴M. J. Rhee and W. W. Destler, Phys. Fluids **17**, 1574 (1974).

⁵W. W. Destler, P. K. Misra, and M. J. Rhee, Phys. Fluids **18**, 1820 (1975).

⁶W. W. Destler, D. W. Hudgings, R. A. Kehs, P. K. Misra, and M. J. Rhee, IEEE Trans. Nuclear Sci. NS-22, 995 (1975).

⁷M. P. Reiser, IEEE Trans. Nuclear Sci. NS-20, 310 (1973).