Millimeter wave emission from a rotating electron ring in a rippled magnetic field

G. Bekefi and R. E. Shefer
Department of Physics and Research Laboratory of Electronics, Massachusetts Institute of Technology, Cambridge, Massachusetts 02139

W. W. Destler
Department of Electrical Engineering, University of Maryland, College Park, Maryland 20742

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We report measurements of millimeter wave emission from a rotating relativistic electron ring (2 MV, 1 kA) in which electrons move in quasi-circular orbits under the combined action of a uniform axial magnetic field and an azimuthally periodic wiggler magnetic field. We observe radiation at frequencies above 91 GHz, at power levels in excess of 200 kW.

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There have been many theoretical1 and experimental2 studies of free-electron lasers (FEL's) in linear geometry with spatially periodic transverse1,2 or longitudinal3-6 magnetic wiggler fields. Such configurations have gain limitations imposed by the finite length of the interaction region. Recently, a novel circular version of the free-electron laser has been explored both theoretically7-9 and experimentally10 in which a rotating, relativistic electron stream is subjected to an azimuthally periodic wiggler field. The potential advantages of circular FEL's as compared with the conventional linear form are several. First, the beam circulates continuously through the wiggler field resulting in a long effective interaction region. Secondly, because of the recirculation of the growing electromagnetic wave, the device provides its own internal feedback and is in essence an oscillator rather than an amplifier, as is the case in linear FEL's. And thirdly, because the electron motion is primarily circular the device is very compact.

There are several ways of producing a rotating relativistic electron stream. One is to subject the electrons to orthogonal electric and magnetic fields as is typical in magnetron-like devices. Here, the electrons undergo a \( \gamma(r) = E_{\parallel}(r) \times B_{\perp} / |B_{\perp}|^2 \) drift in a radial electric field \( \pm E_{\parallel}(r) \) and a uniform axial magnetic field \( \pm B_{\perp} \). Addition of an azimuthally periodic magnetic field \( B_{\parallel}(\theta, r) \) then results in a circular FEL. This scheme has been explored previously,7,8,10 and though the experimental results10 are encouraging, it may have a potential drawback in that the electron velocity \( v(r) \) varies with radial distance \( r \). This velocity shear may lead to degradation of the spectral purity of the emitted electromagnetic radiation, and a reduction in gain and efficiency of the device.

In this letter we describe initial experiments on a circular FEL which uses a monoenergetic rotating electron ring and thereby circumvents the problem of velocity shear mentioned above. Moreover, in the device discussed below one has better control over the circulating current than in a magnetron-like scheme where the anode-cathode gap is part of the magnetic wiggler interaction region.

A high quality (energy spread \( \leq 1\% \)) rotating electron ring is produced by injecting a hollow nonrotating beam into a narrow magnetic cusp.11,12 The hollow beam is generated by field emission from an annular graphite cathode energized by a pulsed, high voltage, high current accelerator (2 MV, 20 kA, 30 ns). The resulting rotating electron ring is guided downstream from the cusp by a uniform axial magnetic field of \( \sim 1.4 \) kG. The ring is 6 cm in radius, has a duration of \( \sim 5 \) ns, and carries an axial current of \( \sim 1.5 \) kA. The electron rotation velocity \( v_\theta \approx 0.96c \), and the electron axial velocity \( v_z \approx 0.2c \). Thus, in the absence of the wiggler magnetic field, the electron orbits form fairly tight helices.
A schematic of the device is illustrated in Fig. 1. It comprises two smooth coaxial stainless steel cylinders of radii \( r_0 = 6.58 \text{ cm} \) and \( r_i = 5.25 \text{ cm} \), respectively. The electron ring propagates within the gap formed by the two cylinders. Superimposed on the axial guiding magnetic field is an azimuthally periodic magnetic wiggler field \( B_w \), which, near the center of the gap, is primarily radial and is thus transverse to the electron flow velocity, as is the case in conventional linear free-electron lasers. A single particle computer simulation program has been generated for the purpose of studying the electron motion in the combined axial and wiggler magnetic fields. We see from Fig. 2 that the trajectory is not perturbed too strongly: it remains quasi-helical, the radial displacements are small, and the electron does not strike the cylinder walls.

In our device, the wiggler magnetic field is produced by an assembly of 384 samarium-cobalt bar magnets, each having a residual induction of \(-9.0 \text{ kG}\). The magnets are positioned behind the grounded stainless steel cylinders and held in place in grooved aluminum holders. To achieve a given periodicity \( l \), the dipole axes of the magnets are arranged as illustrated in Fig. 3. The lower part of the figure shows a Hall-probe measurement of the radial component of the wiggler field at the center of the vacuum gap. The measured field amplitude equals \( 1.31 \text{ kG} \). The axial length of the wiggler is 20 cm. This is achieved by stacking end-to-end four rows of bar magnets. At the present time, all of the radiation measurements described below were made with a wiggler having six spatial periods \( N \) and a periodicity \( l = 6.28 \text{ cm} \). Shorter periodicities are expected to give radiation at frequencies which lie above the range of our detection equipment.

To estimate the radiation frequency we assume that in the presence of the wiggler, the electrons experience a ponderomotive force which causes electron bunching in the \( \theta \) direction. When the \( \theta \)-directed phase velocity \( \omega/(k_w + k_\Theta) \) of this space-charge wave is slightly below the electron velocity \( v_\parallel \), energy can be given up to the electromagnetic wave. Here \( k_w = N/r = 2\pi/l \), \( \omega \) is the radiation frequency; \( k_\Theta = m/r \) is the radiation wave number with \( m \) as the mode number of a transverse magnetic (TM) mode of the coaxial waveguide and \( r = (r_0 + r_i)/2 \). Near cutoff \( (k_\parallel \rightarrow 0) \), one obtains the familiar FEL formula,

\[
\omega \simeq (1 + \beta_\parallel \beta_\gamma)^2 k_w c/K.
\]  

Here \( \beta_\parallel = v_\parallel/c \), \( \gamma = 1 + eV/m_0 c^2 \) with \( V \) as the beam voltage; \( \Omega_w = eB_{\text{ave}}/m_0 \) is the nonrelativistic cyclotron frequency in the wiggler field of amplitude \( B_{\text{ave}} \), and \( K = 1 + \left[\Omega_w(k_w c)^2\right] \).

The radiation generated in the interaction region is al-

FIG. 1. General experimental configuration.

FIG. 2. Calculated particle orbits in the \( r-\theta \) and \( r-z \) planes for an electron injected with \( v_\parallel = 0.20c \), \( v_\perp = 0.96c \) into the interaction space with (a) \( B_{\text{ave}} = 1.4 \text{ kG} \), \( B_{0\perp} = 0 \), and (b) \( B_{\text{ave}} = 1.4 \text{ kG} \), \( B_{0\perp} = 1.3 \text{ kG} \).

FIG. 3. Arrangement of bar magnets (top); Hall probe measurement of the wiggler field at a radial position \( r = 3.92 \text{ cm} \), as a function of azimuthal angle (bottom).
allowed to leak out from the gap formed by the two coaxial cylinders. It is received by means of a small horn antenna, and is guided through various waveguide cut-off filters to a crystal detector where it is rectified and displayed on a fast oscilloscope. Figure 4(c) illustrates the time history of a typical radiation burst at frequencies above 91 GHz as measured with a T-band (91–170 GHz) filter. When the magnetic wiggler field is turned off (by removing the samarium-cobalt magnets from their grooved aluminum cylinders) the emitted power falls to a level too small to be distinguished from background noise [Fig. 4(d)]. We thus conclude that the observed radiation is produced only in the presence of the wiggler field.

We have as yet not addressed the problem of how best to couple out the available radiation. Our horn antenna merely probes the radiation field and receives only a small fraction of the available power. Using the crystal calibration of our detector, the total power radiated from the device at frequencies above 91 GHz is estimated to be no smaller than 200 kW. Inserting experimental parameters into Eq. (1) yields a radiation frequency \( \omega / 2\pi \approx 170 \) GHz. But, we have not yet measured the spectrum.

In addition to the T-band (91–170 GHz) range of frequencies, we also explored emission at lower frequencies, from 21 GHz and up. Here we find that some emission occurs even in the absence of the wiggler magnetic field. The cause of this radiation is the negative mass instability.\(^{16-17}\) However, as a result of the proximity\(^{16}\) of the two grounded, concentric metal cylinders the level of this radiation is greatly reduced compared to that observed in earlier work\(^{14,15}\) on the negative mass instability, where the conducting boundaries were not in such close proximity to the beam. When the wiggler magnetic field is introduced the level of the low-frequency emission remains either unchanged or, in some cases, is diminished. This shows that the presence of the wiggler field does not enhance the negative mass instability, which has been a worrisome possibility.

In conclusion, we have observed radiation in the millimeter wavelength range (\( \lambda < 3.3 \) mm) from a novel type of circular FEL which uses a high quality, high current relativistic electron ring rotating in an azimuthally periodic wiggler magnetic field. The emitted power attributed to the FEL instability is at least 200 kW. Spectral measurements using a calibrated microwave grating spectrometer\(^{10,19}\) will be carried out in the near future. In addition, by rearranging the magnets as illustrated in Fig. 3, we will be able to shorten the wiggler periodicity \( \ell \) and thereby study emission at wavelengths ranging from 0.05 to 1.0 mm.

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