CALCULUS II 1016 – 272 – 05

WINTER 2004 – 05 QUARTER

WORKSHEET # 5

OPTIMIZATION

(1) Suppose that a rectangle is inscribed inside a circle whose area is 72π.

(a) **SKETCH THE DIAGRAM** of the phenomena, and

PART (B) IS ON THE NEXT PAGE

(b) Determine the **MAXIMUM AREA** of the rectangle.

CONTINUE ON THE NEXT PAGE IF NECESSARY

ATTACHED SHEET FOR NUMBER 1

(2) Consider a box with a square base with a missing top whose volume is 25 cubic centimeters. Sketch the diagram and determine the minimum surface area.

(a) **SKETCH THE DIAGRAM** of the phenomena, and

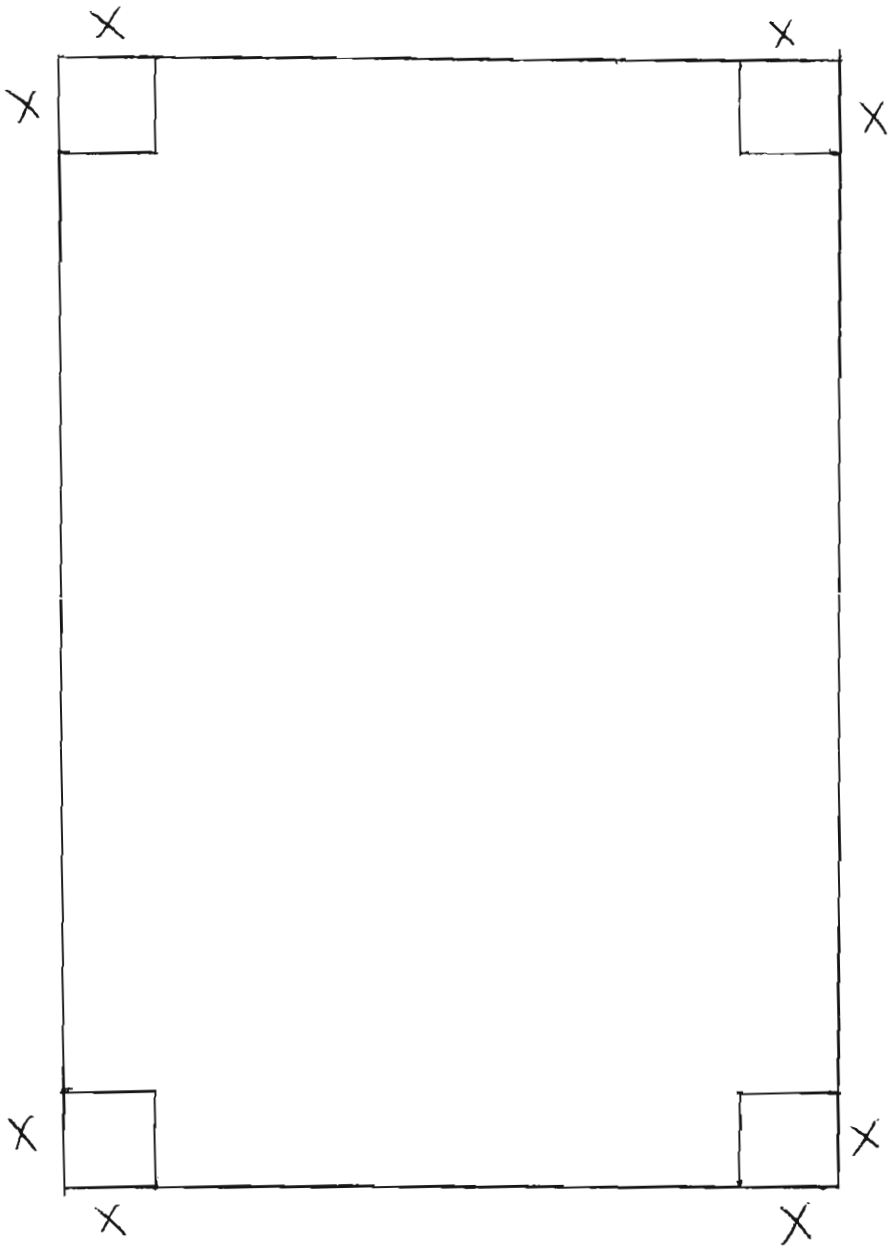
PART (B) IS ON THE NEXT PAGE

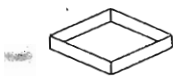
(b) Determine the **MINIMUM VOLUME** of the box.

CONTINUE ON THE NEXT PAGE IF NECESSARY

ATTACHED SHEET FOR NUMBER 2

(3) Consider a rectangular sheet of paper that is 8 x 12 inches in dimentions. Each corner of the rectangle is cut x inches from the edge and then formed into a box as shown in the diagram below:





CONTINUE THE PROBLEM ON THE NEXT PAGE

(a) Determine the Equation that gives the **VOLUME OF THE BOX**, with the restricted interval.

PART (B) IS ON THE NEXT PAGE

(b) From part (a), determine the **MAXIMUM VOLUME** of the box.

PART (C) IS ON THE NEXT PAGE

(c) From parts (a) and (b), determine the **SURFACE AREA** of the box.

(4) Consider a right triangle whose perpendicular sides are 8 x 8 centimeters in dimentions, with an inscribed rectangle in it.

(a) **SKETCH THE DIAGRAM** of the phenomena, and

PART (B) IS ON THE NEXT PAGE

(b) From part (a), determine the Equation that gives the **AREA OF THE RECTANGLE**, with the restricted interval.

PART (C) IS ON THE NEXT PAGE

(c) From part (b), determine the **MAXIMUM AREA** of the rectangle.

CONTINUE ON THE NEXT PAGE IF NECESSARY

ATTACHED SHEET FOR NUMBER 4

CALCULUS III 1016 – 273 – 04

SPRING 2005 QUARTER

WORKSHEET # 7

AREA BETWEEN CURVES

AND

VOLUME BY REVOLUTION

(1) Determine the **AREA**, between the functions

$$y = x^2 - 1 \quad \text{and} \quad y = 7 - x^2 \, .$$

In particular, do the following steps:

(a) **SKETCH** the diagram of the given area. In particular, determine the following:

(i) The dominating function(s).

(ii) The points of intersection(s).

CONTINUE THE PROBLEM ON THE NEXT PAGE

ATTACHED SHEET FOR PART (A)

PART (B) IS ON THE NEXT PAGE

(b) **WRITE** the Definite Integral(s) that describe(s) the given area.

PART (C) IS ON THE NEXT PAGE

(c) **EVALUATE** the integral(s) in part (b).

(2) Determine the **AREA**, between the functions

$$y = x \quad \text{and} \quad y = \sqrt{x+2} \quad \text{on the interval} \quad -2 \leq x \leq 7$$

In particular, do the following steps:

(a) **SKETCH** the diagram of the given area. In particular, determine the following:

(i) The dominating function(s).

(ii) The points of intersection(s).

CONTINUE THE PROBLEM ON THE NEXT PAGE

ATTACHED SHEET FOR PART (A)

PART (B) IS ON THE NEXT PAGE

(b) **WRITE** the Definite Integral(s) that describe(s) the given area.

PART (C) IS ON THE NEXT PAGE

(c) **EVALUATE** the integral(s) in part (b).

(3) Determine the **AREA**, between the functions

$$y = sin(x) \quad \text{and} \quad y = cos(x) \quad \text{on the interval} \quad 0 \leq x \leq 2\pi$$

In particular, do the following steps:

(a) **SKETCH** the diagram of the given area. In particular, determine the following:

(i) The dominating function(s).

(ii) The points of intersection(s).

CONTINUE THE PROBLEM ON THE NEXT PAGE

ATTACHED SHEET FOR PART (A)

PART (B) IS ON THE NEXT PAGE

(b) **WRITE** the Definite Integral(s) that describe(s) the given area.

(4) **AREA** of a RIGHT TRIANGLE.

Consider a right triangle with the following vertices

$$(0,0) \quad , \quad (b,0) \quad \text{and} \quad (b,h)$$

SHOW that:

$$A = \frac{1}{2} * b * h$$

In particular, do the following steps:

(a) **SKETCH** the diagram of the given area. Also, determine

(i) The equations of all the functions

(ii) The dominating functions.

CONTINUE THE PROBLEM ON THE NEXT PAGE

ATTACHED SHEET FOR PART (A)

PART (B) IS ON THE NEXT PAGE

(b) **WRITE** the Definite Integral(s) that describe(s) the given area.

PART (C) IS ON THE NEXT PAGE

(c) **EVALUATE** the integral(s) in part (b).

(5) Determine the **AREA**, between the functions

$$y = \sqrt{x} \quad , \quad y = 2 \quad \text{and} \quad x = 36 \, .$$

In particular, do the following steps:

(a) **SKETCH** the diagram of the given area. In particular, determine the following:

(i) The dominating function(s).

(ii) The points of intersection(s).

CONTINUE THE PROBLEM ON THE NEXT PAGE

ATTACHED SHEET FOR PART (A)

PART (B) IS ON THE NEXT PAGE

(b) **WRITE** the Definite Integral(s) that describe(s) the given area in dx and dy.

PART (C) IS ON THE NEXT PAGE

(c) **EVALUATE** one of the integral(s) in part (b).

(6) **VOLUME** of a SPHERE.

Consider the function

$$y = \sqrt{r^2 - x^2} \quad \text{on the interval} \quad (-r, r)$$

SHOW that:

$$V = \frac{4}{3} * \pi * r^3$$

In particular, do the following steps:

(a) **SKETCH** the diagram of the given area and volume.

PART (B) IS ON THE NEXT PAGE

(b) **WRITE** the Definite Integral(s) that describe(s) the volume.

PART (C) IS ON THE NEXT PAGE

(c) **EVALUATE** the integral(s) in part (b).

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SPRING 2005 QUARTER

WORKSHEET # 8

DIFFERENTIALS,

RIGHT & LEFT HAND SUMS,

AND

GEOMETRICAL INTEGRALS

(1) Using the **differentials**, estimate the following value:

$$\sqrt[3]{63.86}$$

(2) Consider a **CYLINDRICAL PIPE** that is 60 meters long and 80 centimeters in diameter. Due to thermal expansion and shrinking, the walls of the pipe expand and shrink by 1.5 percent and the length of the pipe does not change. Using the differentials:

(a) Approximate the Variation in the **VOLUME OF THE PIPE**.

PART (B) IS ON THE NEXT PAGE

(b) From part (a), determine by how much **Percent**, does the Volume of the pipe shrinks and expands.

PART (C) IS ON THE NEXT PAGE

(c) Approximate the Variation in the **SURFACE AREA OF THE PIPE**.

PART (D) IS ON THE NEXT PAGE

(d) From part (c), determine by how much **Percent**, does the Surface Area of the pipe shrinks and expands.

(3) Consider the **CENTRIPETAL FORCE** of a rotating object on a wheel is given by:

$$F = \frac{mv^2}{r}$$

where

(i) m is **mass** of the object in kilograms.

(ii) v is the **velocity** of the object in meters per second.

(iii) r is the **radius** of the wheel in meters.

Suppose that the object weighs 1500 grams, traveling at 480 meters per minute, and the wheel's is diameter 160 cm. and is off by 1/2 percent. Using the differentials:

CONTINUE THE PROBLEM ON THE NEXT PAGE

(a) Approximate the **Variation** in the **FORCE**.

PART (B) IS ON THE NEXT PAGE

(b) From part (a), determine by how much **Percent**, the **Force** changes.

PART (C) IS ON THE NEXT PAGE

(c) From parts (a) and (b), determine the **Relative Percent** in the error of the **Force Estimation**.

PART (D) IS ON THE NEXT PAGE

(d) From parts (a), (b) and (c), determine the **Percent** of the variation in diameter of the wheel such that the the **Relative Error of the Force Estimation** does not exceed 1 percent.

CONTINUE ON THE NEXT PAGE IF NECESSARY

ATTACHED SHEET FOR PART (D)

(4) Consider the function

$$y = \frac{1}{x^2+1} \quad , \quad 0 \leq x \leq 4$$

(a) **Sketch** the function on the given interval.

PART (B) IS ON THE NEXT PAGE

(b) Write out the **Right and Left Hand Sums** by using $n = 6$.

PART (C) IS ON THE NEXT PAGE

(c) Using part(b), **compare the difference** between the two sums in absolute value.

(5) Consider the function

$$f(x) = \begin{cases} 2x + 6 & \text{for} \quad x < 0\,, \\ 6 - \frac{x}{3} & \text{for} \quad 0 \le x < 6, \\ 4 & \text{for} \quad x \ge 6. \end{cases}$$

(a) **Sketch** the function.

PART (B) IS ON THE NEXT PAGE

(b) Using part (a), **determine the following integrals.**

(i)

$$\int_{-3}^{3} f(x)dx \ .$$

(ii)

$$\int_{0}^{6} f(x)dx \ .$$

CONTINUE THE PROBLEM ON THE NEXT PAGE

(iii)

$$\int_1^{12} f(x)dx \ .$$

(iv)

$$\int_{-5}^{0} f(x)dx \ .$$