ROCHESTER INSTITUTE OF TECHNOLOGY
DEPARTMENT OF MATHEMATICS & STATISTICS

CALCULUS III 1016 – 273 – 03

TEST # 2

APRIL 14TH , 2005

Name:_____

There are **10 questions** on the test. Each question is 15 points each.

YOU MUST SHOW ALL WORK on each problem. You will not receive any credit for any problem done correctly without proper work. In particular, you **MUST SHOW**:

(i) All the steps in solving the Differential Equation.
(ii) All the diagrams must be sketched.
(iii) All the functions must be labeled on each diagram.
(iv) Show all the main equations used and the substitutions.
(v) All the steps in setting up the definite integrals in each problem.
(vi) All the steps of integration.

There are **ABSOLUTELY NO CALCULATORS** allowed on the test.

Good luck. Do not rush through the test.

PROBLEM CHECK LIST:

PUT A CHECK next to the question after you finish it.

QUESTION # 1 _____

QUESTION # 2 _____

QUESTION # 3 _____

QUESTION # 4 _____

QUESTION # 5 _____

QUESTION # 6 _____

QUESTION # 7 _____

QUESTION # 8 _____

QUESTION # 9 _____

QUESTION # 10 _____

(1) Solve the following **INITIAL VALUE PROBLEM**; (15 points):

(i)

$$\frac{dy}{dt} = -3ty^2 .$$

(ii)

$$y(2) = \frac{1}{6}$$

NOTE: You **MUST** check your answer.

CONTINUE ON THE NEXT PAGE IF NECESSARY

ATTACHED SHEET FOR NUMBER 1

(2) Determine the **AREA**, between the following functions; (15 points):

$$y = x^2 - 1 \quad \text{and} \quad y = 7 - x^2 \ .$$

ALL THE STEPS OF THE PROCESS MUST BE SHOWN.

CONTINUE ON THE NEXT PAGE IF NECESSARY

ATTACHED SHEET FOR NUMBER 2

(3) **WRITE** the Definite Integrals the give the **AREA** between the following functions; (15 points):

$$y = x^2 \quad , \quad y = 4x \quad \text{and} \quad y = x + 6 \, .$$

ALL THE STEPS OF THE PROCESS MUST BE SHOWN.

WRITE THE DEFINITE INTEGRALS ONLY; DO NOT EVALUATE.

CONTINUE ON THE NEXT PAGE IF NECESSARY

ATTACHED SHEET FOR NUMBER 3

(4) Consider the following **AREA** between the given functions:

$$y = x + 1 \quad , \quad y = 9 - x \quad \text{and} \quad y = 2 \, .$$

(a) **SKETCH** the given area; (3 points).

CONTINUE THE PROBLEM ON THE NEXT PAGE

(b) **Write** the **DEFINITE INTEGRAL(S)** that descrive the given area in **dx**; (6 points).

WRITE THE DEFINITE INTEGRALS ONLY; DO NOT EVALUATE.

CONTINUE THE PROBLEM ON THE NEXT PAGE

(c) **Write** the DEFINITE INTEGRAL(S) that descrive the given area in **dy**; (6 points).

WRITE THE DEFINITE INTEGRALS ONLY; DO NOT EVALUATE.

(5) Consider the following **AREA** between the given functions:

$$y = \sqrt{x} \quad , \quad y = 1 \quad \text{and} \quad x = 16 \, .$$

(a) **SKETCH** the given area; (3 points).

CONTINUE THE PROBLEM ON THE NEXT PAGE

(b) **Write** the **DEFINITE INTEGRAL(S)** that descrive the given area in **dx**; (6 points).

WRITE THE DEFINITE INTEGRALS ONLY; DO NOT EVALUATE.

CONTINUE THE PROBLEM ON THE NEXT PAGE

(c) **Write** the **DEFINITE INTEGRAL(S)** that descrive the given area in **dy**; (6 points).

WRITE THE DEFINITE INTEGRALS ONLY; DO NOT EVALUATE.

(6) Consider the following **AREA** between the given functions:

$$y = 4x - x^3 \quad \text{and the x-axis} \quad x \geq 0 \, .$$

(a) **SKETCH** the given area; (3 points).

CONTINUE THE PROBLEM ON THE NEXT PAGE

(b) **WRITE** the **DEFINITE INTEGRAL(S)** that gives the **VOLUME** by rotating the area about the **x-axis**; (6 points).

WRITE THE DEFINITE INTEGRALS ONLY; DO NOT EVALUATE.

CONTINUE THE PROBLEM ON THE NEXT PAGE

(c) **WRITE** the **DEFINITE INTEGRAL(S)** that gives the **VOLUME** by rotating the area about the **y-axis**; (6 points).

WRITE THE DEFINITE INTEGRALS ONLY; DO NOT EVALUATE.

(7) **DETERMINE** the following **VOLUMES**:

(a) By rotating the the function

$$y = \frac{1}{x} \quad \text{on the interval} \quad \frac{1}{4} \leq x \leq \frac{1}{2}$$

about the **x-axis**; (7.5 points).

MUST SHOW ALL THE STEPS OF THE PROCESS.

CONTINUE ON THE NEXT PAGE IF NECESSARY

ATTACHED SHEET FOR PART(A)

CONTINUE THE PROBLEM ON THE NEXT PAGE

(b) By rotating the the function

$$y = \frac{1}{x} \quad \text{on the interval} \quad \frac{1}{4} \leq x \leq \frac{1}{2}$$

about the **y-axis**; (7.5 points).

MUST SHOW ALL THE STEPS OF THE PROCESS.

CONTINUE ON THE NEXT PAGE IF NECESSARY

ATTACHED SHEET FOR PART(B)

(8) **WRITE** the **DEFINITE INTEGRALS** that give the following **VOLUMES**:

(a) By rotating the the function

$$y = \frac{x}{4} \quad \text{on the interval} \quad 1 \leq y \leq 3$$

about the line **y=-2**; (7.5 points).

MUST SHOW ALL THE STEPS OF THE PROCESS.

CONTINUE THE PROBLEM ON THE NEXT PAGE

(b) By rotating the the function

$$y = \frac{x}{4} \quad \text{on the interval} \quad 1 \leq y \leq 3$$

about the line x=4; (7.5 points).

MUST SHOW ALL THE STEPS OF THE PROCESS.

(9) **WRITE** the **DEFINITE INTEGRAL(S)** that give the **PERIMETER** of the polygon bounded between the following functions:

$$y = x^2 \quad \text{and} \quad y = 8 - x^2 \ .$$

ALL THE STEPS OF THE PROCESS MUST BE SHOWN; (15 points).

CONTINUE ON THE NEXT PAGE IF NECESSARY

ATTACHED SHEET FOR NUMBER 9

(a) **SKETCH** the diagram of the given area, volume and surface area; (3 points).

CONTINUE THE PROBLEM ON THE NEXT PAGE

(b) **WRITE** the Definite Integral(s) that describe(s) the surface area; (8 points).

CONTINUE THE PROBLEM ON THE NEXT PAGE

(c) **EVALUATE** the integral(s) in part (b); (4 points).

ROCHESTER INSTITUTE OF TECHNOLOGY
DEPARTMENT OF MATHEMATICS & STATISTICS

CALCULUS II 1016 – 272 – 03

TEST # 2

APRIL 14TH , 2005

Name:_____

There are **8 questions** on the test. Each question is 18 points each.

YOU MUST SHOW ALL WORK on each problem. You will not receive any credit for any problem done correctly without proper work. In particular, you **MUST SHOW**:

(i) All the diagrams when doing word problems.
(ii) All the steps of differentiation.
(iii) All the steps in rewriting the functions if necessary before differentiation.
(iv) All the steps in setting the derivative(s) equal to 0.
(v) All the steps in determining the sign of the derivatives.
(vi) All the steps in sketching the functions and labeling all the properties.
(vii) All the steps in summations and integration.

There are **ABSOLUTELY NO CALCULATORS** allowed on the test.

PROBLEM CHECK LIST:

PUT A CHECK next to the question after you finish it.

QUESTION # 1 _____

QUESTION # 2 _____

QUESTION # 3 _____

QUESTION # 4 _____

QUESTION # 5 _____

QUESTION # 6 _____

QUESTION # 7 _____

QUESTION # 8 _____

(1) **SKETCH** the following function:

$$y = 2x^2 - x^4 \qquad \text{on the interval} \qquad -2 \leq x \leq 3.$$

You **MUST** determine the following properties and indicate these properties on the graph:

(a) All the **END POINTS**; (1 point).

(b) All the **CRITICAL POINTS**; (2 points).

CONTINUE THE PROBLEM ON THE NEXT PAGE

(c) Which critical points are:

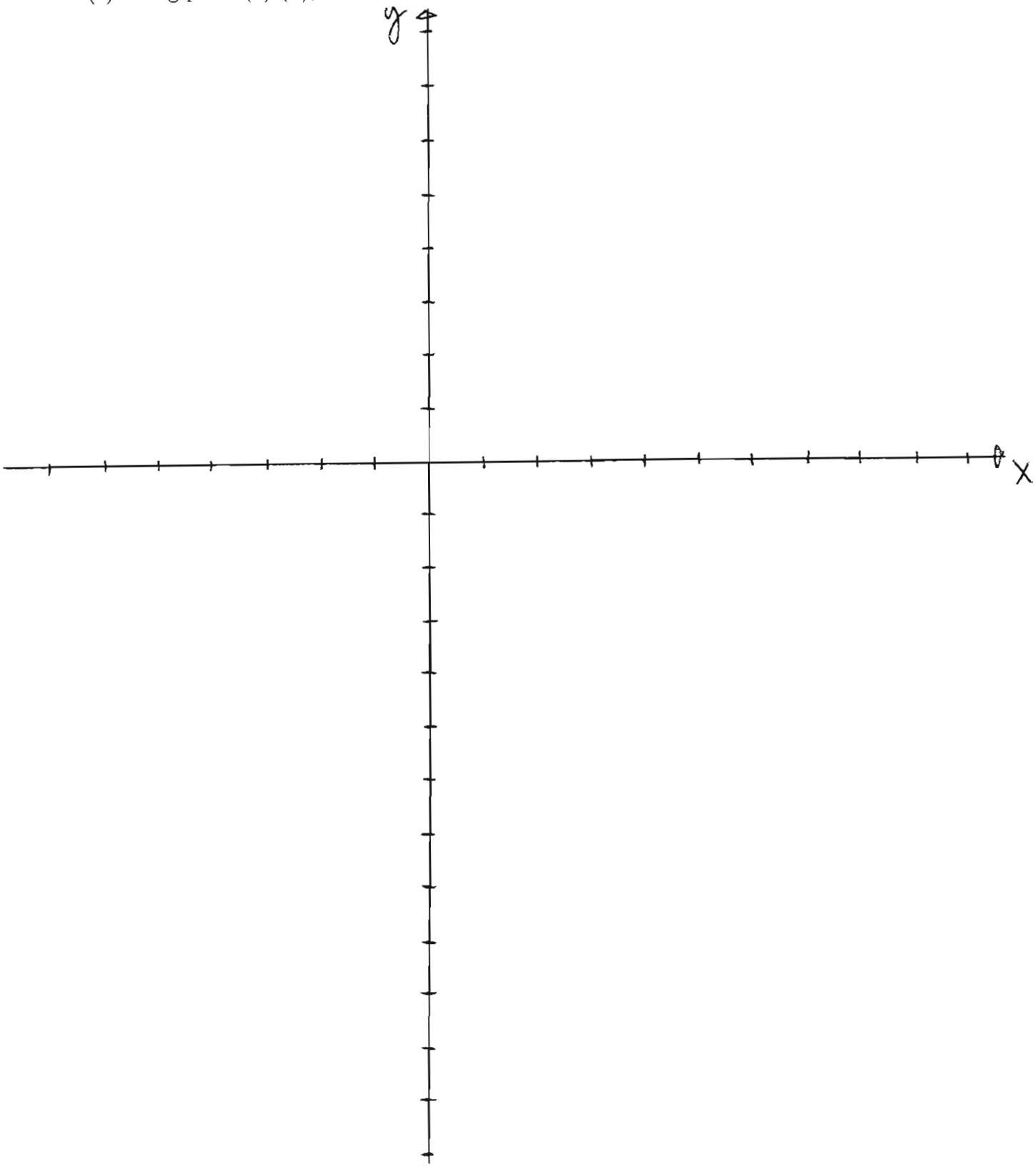
(i) **RELATIVE MINIMUM**; (1 point).

(ii) **RELATIVE MAXIMUM**; (1 point).

(d) All the **INFLECTION POINTS**; (1 point).

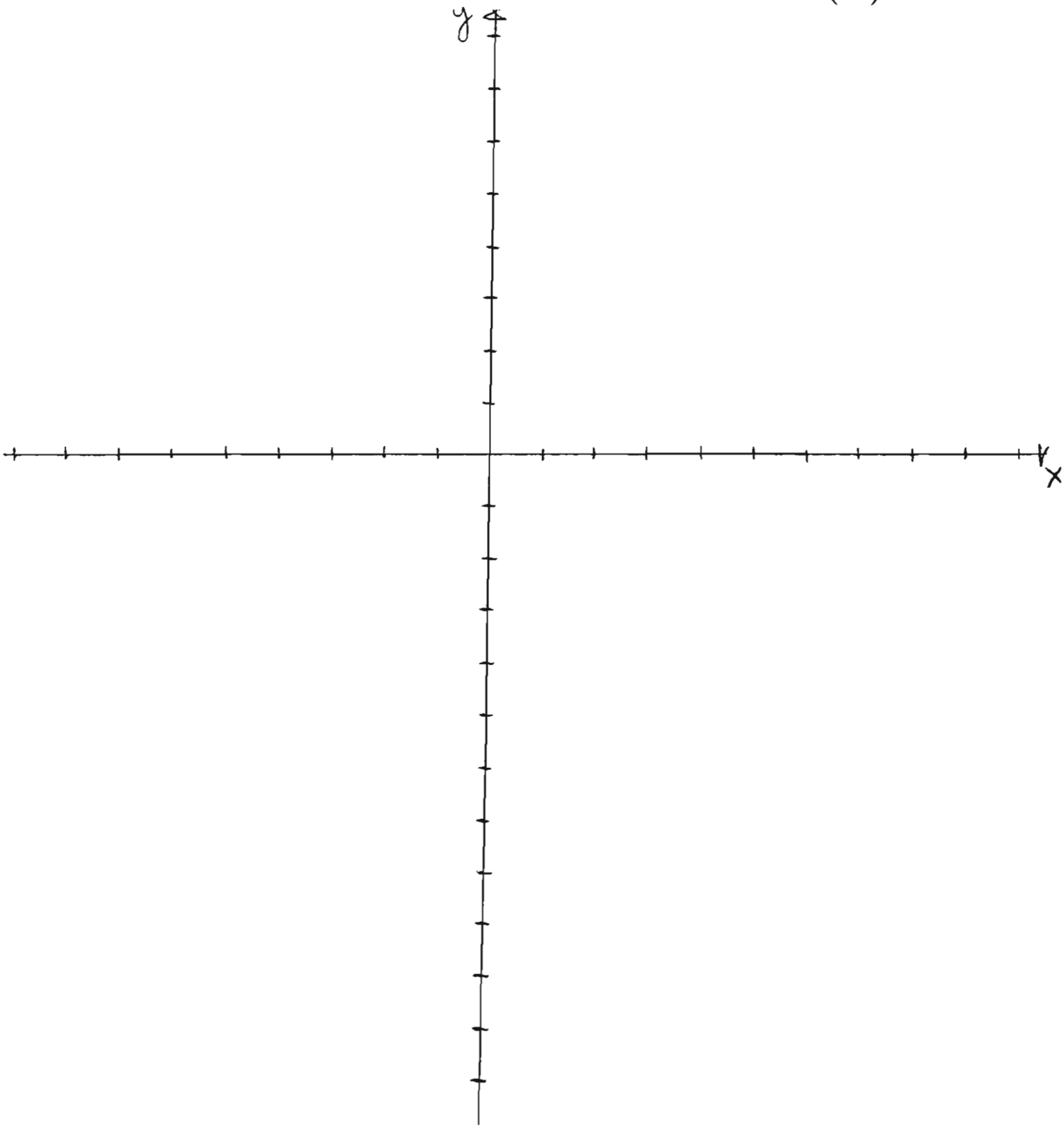
CONTINUE THE PROBLEM ON THE NEXT PAGE

(e) Using parts (a)-(d), **SKETCH** the function below; (10 points):



CONTINUE THE PROBLEM ON THE NEXT PAGE

ATTACHED SHEET FOR PART (E)



CONTINUE THE PROBLEM ON THE NEXT PAGE

(i) Using part (e), determine the **ABSOLUTE MIMUMIM**; (1 point).

(ii) Using part (e), determine the **ABSOLUTE MAXIMUM**; (1 point).

USE THE GEOMETRY FORMULA SHEET

(2) Suppose that a **RECTANGLE** is inscribed inside a circle whose circumference is 8π.

(a) **SKETCH THE DIAGRAM** of the phenomena; (4 points).

CONTINUE THE PROBLEM ON THE NEXT PAGE

(b) Determine the **MAXIMUM AREA** of the RECTANGLE; (14 points).

MUST SHOW ALL THE WORK AND DETAILS

CONTINUE THE PROBLEM ON THE NEXT PAGE

ATTACHED SHEET FOR PART (B)

USE THE GEOMETRY FORMULA SHEET

(3) Consider a **BOX** with a square base with a missing top whose volume is 32 cubic inches.

(a) **SKETCH THE DIAGRAM** of the phenomena; (4 points).

CONTINUE THE PROBLEM ON THE NEXT PAGE

(b) Determine the **MINIMUM SURFACE AREA** of the BOX; (14 points).

MUST SHOW ALL THE WORK AND DETAILS

CONTINUE THE PROBLEM ON THE NEXT PAGE

ATTACHED SHEET FOR PART (B)

(4) Using the **DIFFERENTIALS**, estimate the following value: (18 points);

$$\sqrt{36.1}$$

WRITE THE EXPESSION ONLY IN TERMS OF DIFFERENTIALS.

USE THE GEOMETRY FORMULA SHEET

(5) Consider a **CYLINDRICAL PIPE** that is 20 meters long and 40 centimeters in diameter. Due to thermal expansion and shrinking, the walls of the pipe expand and shrink by 1 percent.

Using the **Differentials**, determine the **RELATIVE ERROR** in the **volume** of the pipe; (18 points).

MUST SHOW ALL THE WORK AND DETAILS

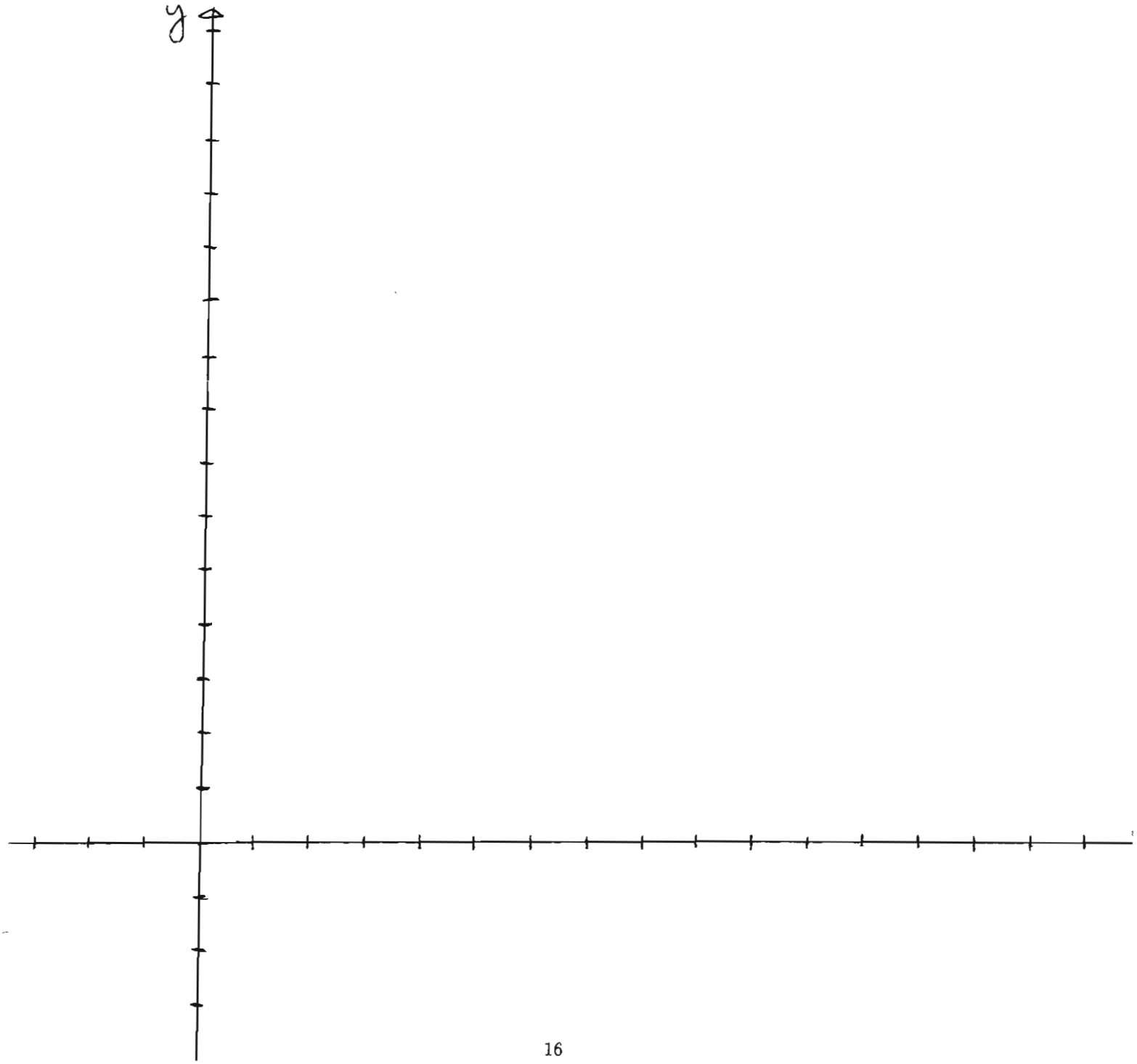
CONTINUE THE PROBLEM ON THE NEXT PAGE

ATTACHED SHEET FOR NUMBER 5

(6) Consider the function

$$f(x) = x^2 \quad , \quad 1 \leq x \leq 5$$

(a) **Sketch** the function on the given interval.



CONTINUE THE PROBLEM ON THE NEXT PAGE

(b) Write out the **RIGHT HAND SUM** for $n = 8$; (9 points).

MUST SHOW ALL THE DETAILS;

DO AS MUCH AS YOU CAN WITHOUT A CALCULATOR.

CONTINUE THE PROBLEM ON THE NEXT PAGE

(c) Write out the **LEFT HAND SUM** for $n = 8$; (9 points).

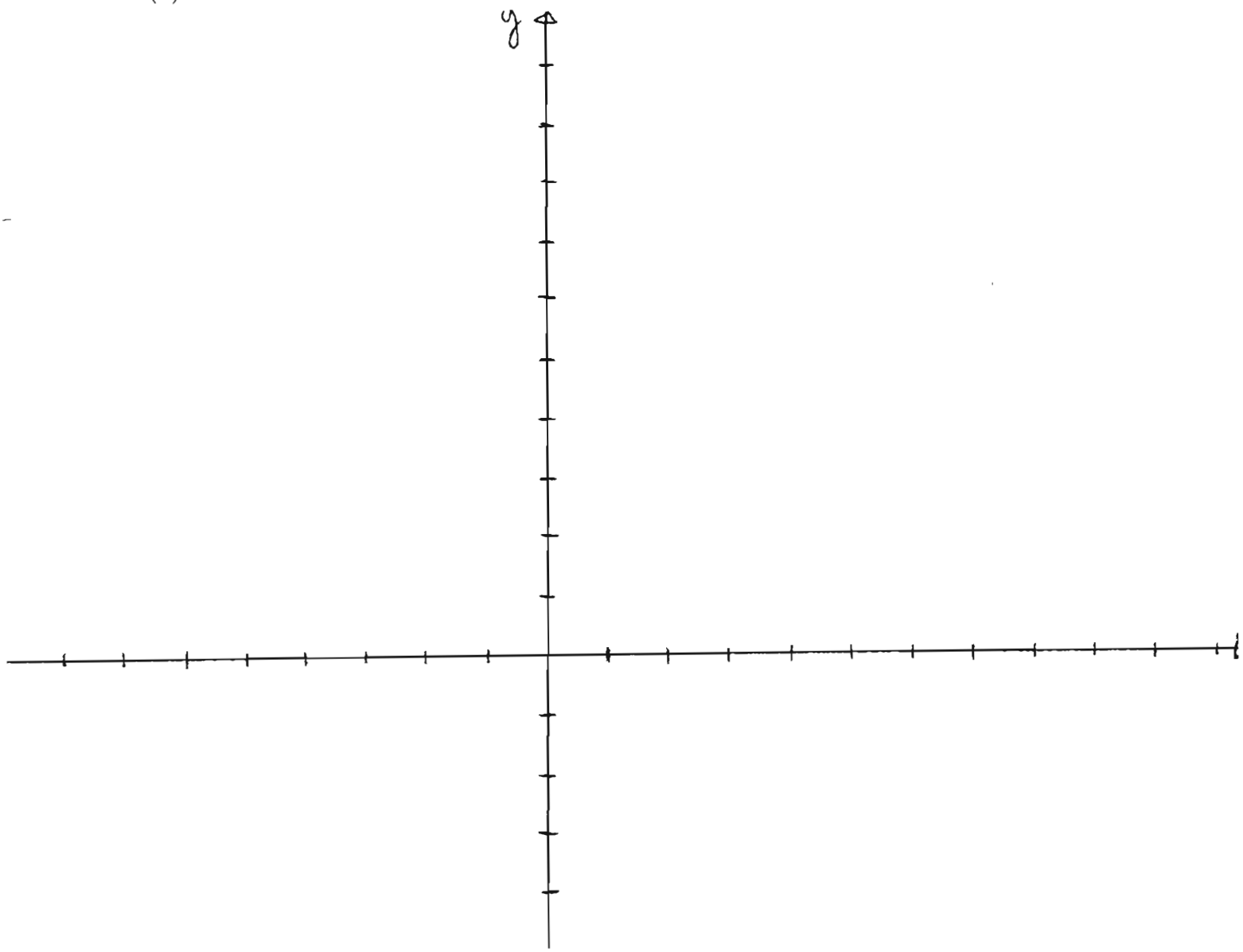
MUST SHOW ALL THE DETAILS;

DO AS MUCH AS YOU CAN WITHOUT A CALCULATOR.

(7) Consider the function

$$f(x) = \begin{cases} 2x + 8 & \text{for} \quad x < 0\,, \\ 8 - \frac{\pi}{2} & \text{for} \quad 0 \le x < 8, \\ 4 & \text{for} \quad x \ge 8. \end{cases}$$

(a) **SKETCH** the function.



CONTINUE THE PROBLEM ON THE NEXT PAGE

(b) Using part (a), **DETERMINE** the following integrals **GEOMETRICALLY**.

(i) (6 points)

$$\int_{-4}^{0} f(x)dx \ .$$

MUST SHOW ALL THE WORK

CONTINUE THE PROBLEM ON THE NEXT PAGE

(ii) (6 points)

$$\int_0^{16} f(x)dx \ .$$

MUST SHOW ALL THE WORK

CONTINUE THE PROBLEM ON THE NEXT PAGE

(iii) (6 points)

$$\int_{-2}^{8} f(x)dx \ .$$

MUST SHOW ALL THE WORK

(8) Using the **SUMMATION FORMULAS** on the last page, determine; (6 points each):

(a) **Express** the following sum in the SIGMA notation and then **simplify the sum**:

$$3 + 6 + 9 + 12 + \ldots + 270 .$$

MUST SHOW ALL THE STEPS AND THE USE OF FORMULA(S).

CONTINUE THE PROBLEM ON THE NEXT PAGE

(b) **Express** the following sum in the SIGMA notation and then **simplify the sum**:

$$4 + 16 + 36 + 64 + \ldots + 256 .$$

MUST SHOW ALL THE STEPS AND THE USE OF FORMULA(S).

CONTINUE THE PROBLEM ON THE NEXT PAGE

(c) Simplify the following sum:

$$\sum_{k=11}^{40} (2k+3) \ .$$

MUST SHOW ALL THE STEPS AND THE USE OF FORMULA(S).

CONTINUE THE PROBLEM ON THE NEXT PAGE

GEOMETRY FORMULA SHEET

(a) **CIRCLE**

$$\mathbf{A} = \pi r^2$$
$$\mathbf{C} = 2\pi r$$

(b) **RECTANGLE**

$$\mathbf{A} = lw$$
$$\mathbf{P} = 2l + 2w$$
$$d^2 = l^2 + w^2$$

(c) **BOX**

$$\mathbf{V} = lwh$$
$$\mathbf{S} = 2lw + 2lh + +2wh$$

(d) **CYLINDER**

$$\mathbf{V} = \pi r^2 h$$
$$\mathbf{S} = 2\pi rh + 2\pi r^2$$

SUMMATION FORMULAS

(a)

$$\sum_{k=1}^{n} k = \frac{n(n+1)}{2}$$

(b)

$$\sum_{k=1}^{n} k^2 = \frac{n(n+1)(2n+1)}{6}$$

(c)

$$\sum_{k=1}^{n} c\, a_k = c \sum_{k=1}^{n} a_k$$

(d)

$$\sum_{k=1}^{n} [a_k + b_k] = \sum_{k=1}^{n} a_k + \sum_{k=1}^{n} b_k$$

ROCHESTER INSTITUTE OF TECHNOLOGY
DEPARTMENT OF MATHEMATICS & STATISTICS

CALCULUS I 1016 – 271 – 06

TEST # 2

OCTOBER 14TH , 2004

Name:_____

There are **7 questions** on the test. Each question will be **21 points each**.

YOU MUST SHOW ALL WORK on each problem. You will not receive any credit for any problem done correctly without proper work. In particular, you **MUST SHOW**:

(i) All the algebra steps in limits.
(ii) All the properties of limits that you are using.
(iii) Explain why a limit does not exists.
(iv) Show all the necessary steps of substitution.
(v) Draw diagram(s) in word problems and show all the substitutions.
(vi) Explain why a function would fail to be continuous.

There are **ABSOLUTELY NO CALCULATORS** allowed on the test.

Good luck. Do not rush through the test.

(1) Consider a circle with a fixed diameter of 6 meters.

In addition, a **Rectangle is inscribed** inside the circle.

Sketch the diagram and then write the **AREA OF THE RECTANGLE** as a function of one variable. (21 points)

(2) Consider the function:

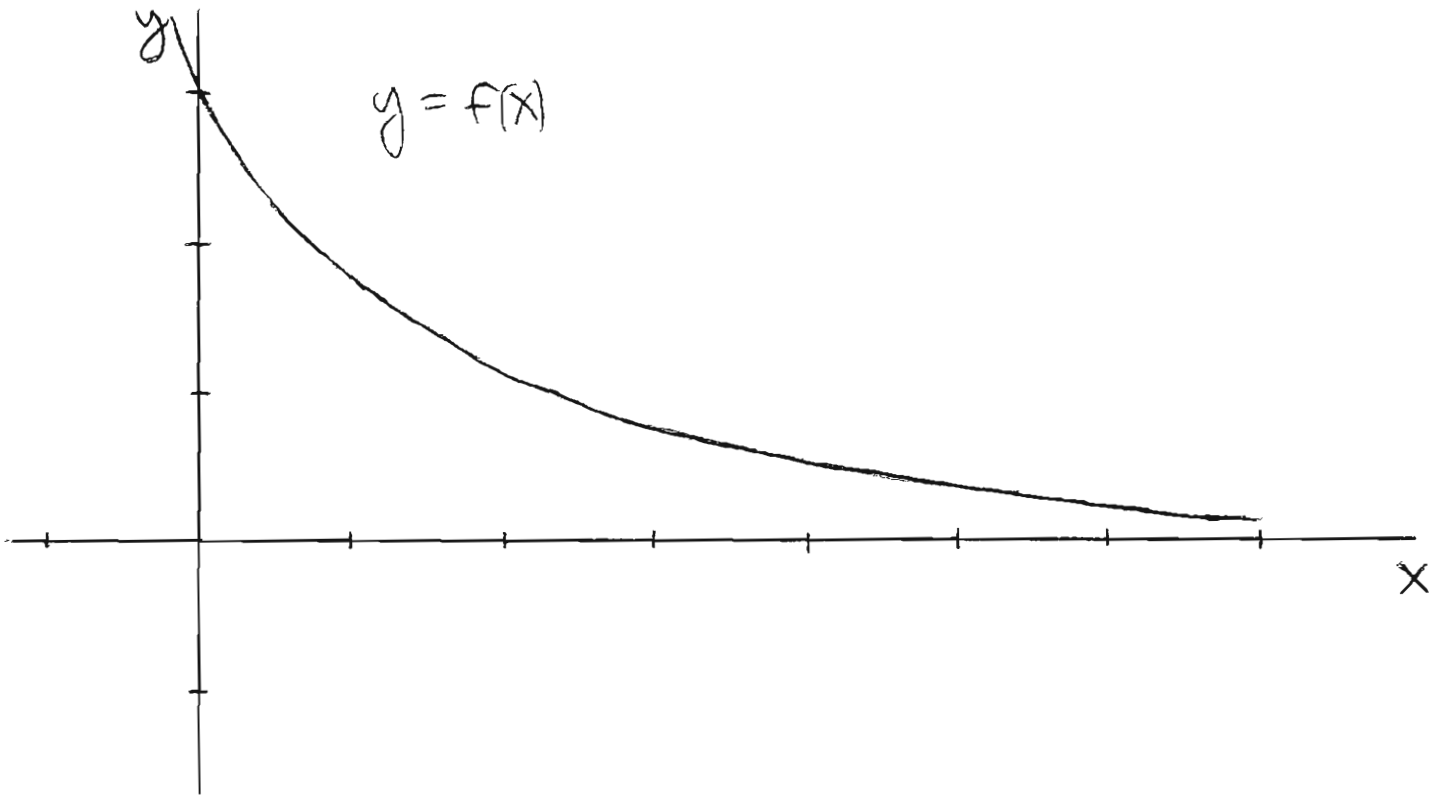
$$f(x) = \sqrt{x}$$

Determine the following:

(a) The **SECANT SLOPE** between $x = 1$ and $x = 9$; (10 points).

(b) Using part (a), write the **EQUATION OF THE SECANT LINE** between $x = 1$ and $x = 9$; (11 points).

(3) Consider the function:



Determine the following:

(a) The **TANGENT SLOPE** at $x = 2$; (14 points).

PART (B) IS ON THE NEXT PAGE

(b) Using part (a), write the **EQUATION OF THE TANGENT LINE** at $x = 2$; (7 points).

(4) Determine the following **limits**; (7 points each):

(a)

$$\lim_{x \to 1} \frac{x-1}{x^2-1}$$

(b)

$$\lim_{x \to 9} \frac{x-9}{3-\sqrt{x}}$$

PART (C) IS ON THE NEXT PAGE

(c)

$$\lim_{x \to 0} \frac{sin^2(x)}{x}$$

(5) Consider the following function:

$$f(x) = \begin{cases} 2x + 1 & \text{if } x \leq 1\,, \\ \\ x^2 - 1 & \text{if } x > 1. \end{cases}$$

Sketch the function above and determine the following **limits** and **properties**:

(a) (3 points)

$$\lim_{x \to -1} f(x)$$

(b) (3 points)

$$\lim_{x \to 3} f(x)$$

PARTS (C), (D), (E) AND (F) ARE ON THE NEXT PAGE

(c) (4 points)

$$\lim_{x \to 1^+} f(x)$$

(d) (4 points)

$$\lim_{x \to 1^-} f(x)$$

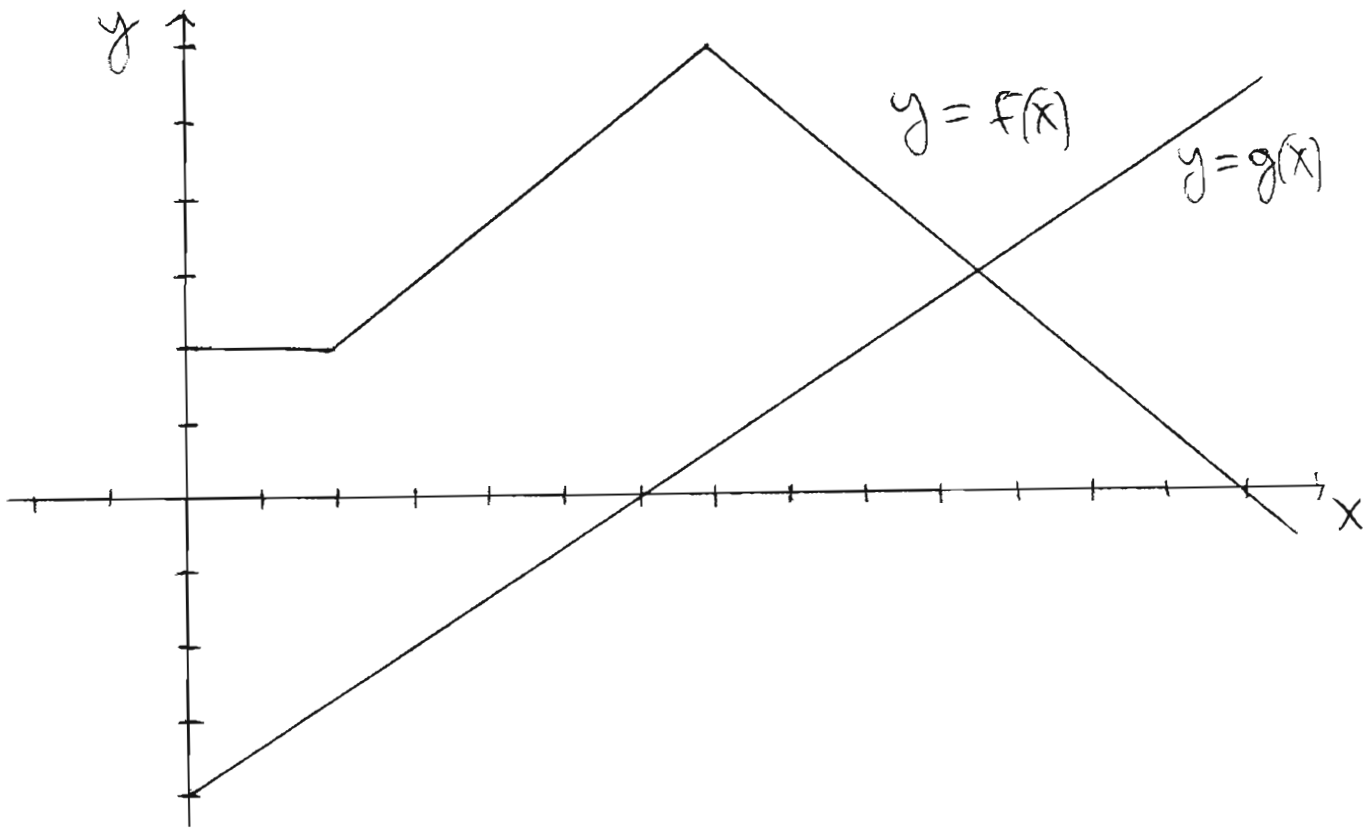
(e) (4 points)

$$\lim_{x \to 1} f(x)$$

(f) If f(x) is continuous at $x = 1$; (3 points).

NOTE: Must explain in limits why or why not.

(6) Consider two functions f(x) anf g(x) described by the following graph:



Determine if the following limits exist. If so, determine them, if not, explain why:

NOTE: You must show the use of the limit properties.

(a) (3 points)

$$\lim_{x \to 3} f(x)$$

(b) (3 points)

$$\lim_{x \to 3} g(x)$$

PARTS (C), (D), AND (E) ARE ON THE NEXT PAGE

(c) (5 points)

$$\lim_{x \to 5} [f(x) - 4 * g(x)]$$

(d) (5 points)

$$\lim_{x \to 1} \left[\frac{4 * f(x)}{3 + g(x)} \right]$$

(e) (5 points)

$$\lim_{x \to 6} f[g(x)]$$

PART (B) IS ON THE NEXT PAGE

(b) By using part(a), determine the **EQUATION OF THE TANGENT LINE** at $x = 2$ (11 points).

ROCHESTER INSTITUTE OF TECHNOLOGY
DEPARTMENT OF MATHEMATICS & STATISTICS

CALCULUS II 1016 – 272 – 05

TEST # 1

DECEMBER 17TH, 2004

Name:_____

There are **7 questions** on the test. Each question is 20 points each.

YOU MUST SHOW ALL WORK on each problem. You will not receive any credit for any problem done correctly without proper work. In particular, you **MUST SHOW**:

(i) All the steps of differentiation.
(ii) All the method(s) of differentiation you are using.
(iii) All the algebra in determining the sign of the derivative(s).
(iv) Show all the work with limits.
(v) Show all the necessary properties of a function on the graph such as critical and inflection point(s).

There are **ABSOLUTELY NO CALCULATORS** allowed on the test.

Good luck. Do not rush through the test.

(1) Consider the function:

$$y = \frac{x}{x^2 + 8}$$

Using the **FIRST DERIVATIVE TEST**, determine the following properties of the above function:

(a) All the **CRITICAL POINTS**; (10 points).

PART (B) IS ON THE NEXT PAGE

(b) Which critical points are:

(i) **Relative Maximum**; (5 points).

(ii) **Relative Minimum**; (5 points).

(2) Consider the function:

$$y = \frac{1}{x^2 + 1}$$

DETERMINE the following properties of the above function:

(a) All the **INFLECTION POINTS**; (10 points).

PART (B) IS ON THE NEXT PAGE

(b) The interval(s) where:

(i) The function is **Concave up**; (5 points).

(ii) The function is **Concave down**; (5 points).

(3) Consider the function:

$$y = x^4 - 8x^2$$

Using the **SECOND DERIVATIVE TEST**, determine which critical points of the function are a:

(a) **Relative Maximum**; (10 points).

PART (B) IS ON THE NEXT PAGE

(b) **Relative Minimum**; (10 points).

(4) Consider the **HEIGHT OF THE CANNON BALL** described by the following function:

$$h(t) \;=\; 96t \;-\; 16t^2 \quad , \quad t \geq 0 \; .$$

where:

(1) t is time in seconds, and

(2) h(t) is the height of the cannon ball in feet.

Determine the **MAXIMUM HEIGHT OF THE CANNON BALL**; (20 points).

NOTE: You **MUST** verify that it is a relative maximum in terms of first and/or second derivative.

CONTINUE ON THE NEXT PAGE IF NECESSARY

ATTACHED SHEET FOR NUMBER 4

(5) Consider the **PERIMETER OF THE RECTANGLE** described by the following function:

$$P(l) = 2l + \frac{16}{l} \quad , \quad l > 0$$

where:

(1) l is the Length of the rectangle in centimeters, and

(2) P(l) is the Perimeter of the rectangle in centimeters.

Determine the **MINIMUM PERIMETER** of the rectangle; (20 points).

NOTE: You **MUST** verify that it is a relative minimum in terms of first and/or second derivative.

CONTINUE THE PROBLEM ON THE NEXT PAGE

ATTACHED SHEET FOR NUMBER 5

(6) Determine the following **LIMITS**:

(a) (10 points)

$$\lim_{x \to \infty} \left(x^4 + 3x - 5x^5 \right)$$

(b) (10 points)

$$\lim_{x \to \infty} \frac{x^2}{(1 - 2x)^2}$$

(7) **SKETCH** the following function; (20 points):

$$y = x^3 - 12x \ .$$

You **MUST** determine the following properties and indicate these properties on the graph:

(a) All the **CRITICAL POINTS**.

(b) Which critical points are:

(i) **Relative Maximum**.

(ii) **Relative Minimum**.

(c) All the **INFLECTION POINTS**.

(d)

$$\lim_{x \to \infty} \left(x^3 - 12x \right) \ .$$

(e)

$$\lim_{x \to -\infty} \left(x^3 - 12x \right) \ .$$

CONTINUE THE PROBLEM ON THE NEXT PAGE

ATTACHED SHEET FOR NUMBER 7

CONTINUE ON THE NEXT PAGE IF NECESSARY

ATTACHED SHEET FOR NUMBER 7

CONTINUE ON THE NEXT PAGE IF NECESSARY

ATTACHED SHEET FOR NUMBER 7

ROCHESTER INSTITUTE OF TECHNOLOGY
DEPARTMENT OF MATHEMATICS & STATISTICS

CALCULUS III – C 1016 – 273 – 03

TEST # 3

MAY 5th , 2005

Name:_____

There are **10 questions** on the test. Each question is 15 points each.

YOU MUST SHOW ALL WORK on each problem. You will not receive any credit for any problem done correctly without proper work. In particular, you **MUST SHOW**:

(i) All the steps in Integration; especially, splitting into components.
(ii) All the components of each integral must be identified.
(iii) All the method(s) of each integral must be identified.
(iv) Diagrams must be sketched when doing word problems with work.
(v) Diagrams must be sketched when doing problems with center of mass.
(vi) All the steps of substitution (trigonometric identities) and integration.

There are **ABSOLUTELY NO CALCULATORS** allowed on the test.

Good luck. Do not rush through the test.

PROBLEM CHECK LIST:

PUT A CHECK next to the question after you finish it.

QUESTION # 1 _____

QUESTION # 2 _____

QUESTION # 3 _____

QUESTION # 4 _____

QUESTION # 5 _____

QUESTION # 6 _____

QUESTION # 7 _____

QUESTION # 8 _____

QUESTION # 9 _____

QUESTION # 10 _____

USE THE GENERAL FORMULAS AT THE END THE TEST

(1) Determine the **CENTER OF MASS** of the triangle with the following vertices:

$$(0,0) \quad , \quad (b,0) \quad \text{and} \quad (b,h)$$

MUST SHOW ALL THE STEPS OF THE PROCESS; (15 points).

CONTINUE ON THE NEX PAGE IF NECESSARY

ATTACHED SHEET FOR NUMBER 1

USE THE GENERAL FORMULAS AT THE END THE TEST

(2) Consider a **SPRING** that sustains 35 Netwons of force for every 50 centimeters. Furthermore, the spring's natural length at rest is 40 centimeters. **WRITE** the following definite integrals only that give:

(a) The amount of **WORK** it will take to **STRETCH** the spring from 56 to 64 centimeters; (7.5 points).

MUST SHOW ALL THE STEPS OF THE PROCESS.

PART (B) IS ON THE NEXT PAGE

(b) The amount of **WORK** it will take to **COMPRESS** the spring from 36 to 20 centimeters; (7.5 points).

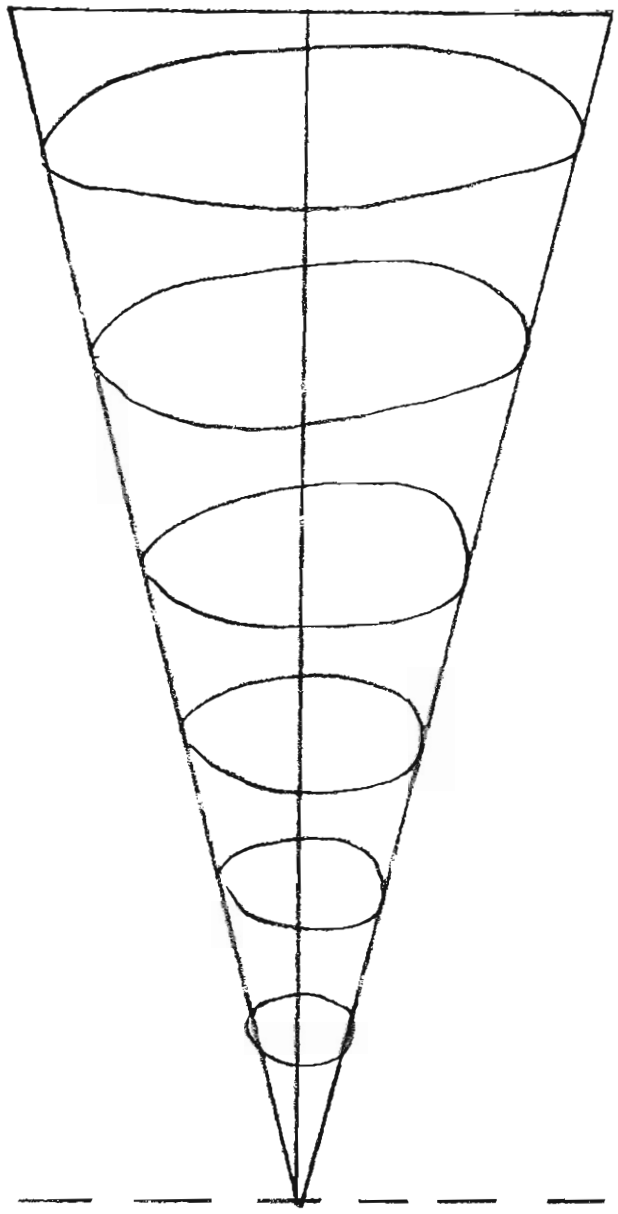
MUST SHOW ALL THE STEPS OF THE PROCESS.

USE THE GENERAL FORMULAS AT THE END THE TEST

(3) Consider a **CONICAL TANK** that is 8 feet long and 1 foot in diameter as shown in the diagram below. Using that the density of water is $\rho = 62.5$ and $g = 16$ in customary units, **WRITE** the following definite integral only that gives:

The amount of **WORK** it will take to **FILL** the tank completely. (15 points)

MUST SHOW ALL THE STEPS OF THE PROCESS.



CONTINUE THE PROBLEM ON THE NEXT PAGE

ATTACHED SHEET FOR NUMBER 3

CONTINUE ON THE NEX PAGE IF NECESSARY

ATTACHED SHEET FOR NUMBER 3

USE THE GENERAL FORMULAS AT THE END THE TEST

(4) **DETERMINE** the following **INTEGRAL**: (15 points);

$$\int 2tan^{-1}(x)\ dx\ .$$

MUST SHOW ALL THE STEPS OF THE PROCESS.

CONTINUE ON THE NEX PAGE IF NECESSARY

ATTACHED SHEET FOR NUMBER 4

USE THE GENERAL FORMULAS AT THE END THE TEST

(5) **DETERMINE** the following **INTEGRAL**: (15 points);

$$\int e^{2t} cos(2t)\ dt\ .$$

MUST SHOW ALL THE STEPS OF THE PROCESS.

CONTINUE ON THE NEXT PAGE IF NECESSARY

ATTACHED SHEET FOR NUMBER 5

USE THE GENERAL FORMULAS AT THE END THE TEST

(6) **DETERMINE** the following **INTEGRAL**: (15 points);

$$\int 2x\ Ln(x+1)\ dx\ .$$

MUST SHOW ALL THE STEPS OF THE PROCESS.

CONTINUE ON THE NEXT PAGE IF NECESSARY

ATTACHED SHEET FOR NUMBER 6

USE THE GENERAL FORMULAS AT THE END THE TEST

(7) **DETERMINE** the following **INTEGRAL**: (15 points);

$$\int \frac{x^2}{x^2+1}\, dx \ .$$

MUST SHOW ALL THE STEPS OF THE PROCESS.

CONTINUE ON THE NEXT PAGE IF NECESSARY

ATTACHED SHEET FOR NUMBER 7

USE THE GENERAL FORMULAS AT THE END THE TEST

(8) **DETERMINE** the following **INTEGRAL**: (15 points);

$$\int \frac{4}{x^3 - x}\, dx \ .$$

MUST SHOW ALL THE STEPS OF THE PROCESS.

CONTINUE ON THE NEXT PAGE IF NECESSARY

ATTACHED SHEET FOR NUMBER 8

USE THE GENERAL FORMULAS AT THE END THE TEST

(9) **DETERMINE** one of the following **INTEGRALS**: (15 points);

$$\int tan^3(\theta)\, d\theta \qquad \text{or} \qquad \int cos^3(\theta)\, d\theta.$$

MUST SHOW ALL THE STEPS OF THE PROCESS.

CONTINUE THE PROBLEM ON THE NEXT PAGE

ATTACHED SHEET FOR NUMBER 9

USE THE GENERAL FORMULAS AT THE END THE TEST

(10) **DETERMINE** one of the following **INTEGRALS**: (15 points);

$$\int csc^4(\theta)\, d\theta \qquad \text{or} \qquad \int \frac{sin(\theta)}{cos^3(\theta)}\, d\theta.$$

MUST SHOW ALL THE STEPS OF THE PROCESS.

CONTINUE THE PROBLEM ON THE NEXT PAGE

ATTACHED SHEET FOR NUMBER 10

TRIGONOMETRIC IDENTITIES:

(i)
$$sin^2(\theta) + cos^2(\theta) = 1$$

(ii)
$$tan^2(\theta) + 1 = sec^2(\theta)$$

(iii)
$$cot^2(\theta) + 1 = csc^2(\theta)$$

(iv)
$$sin(2\theta) = 2sin(\theta)cos(\theta)$$

(v)
$$sin^2(\theta) = \frac{1 - cos(2\theta)}{2}$$

(vi)
$$cos^2(\theta) = \frac{1 + cos(2\theta)}{2}$$

ADDITIONAL FORMULAS:

(i) **INTEGRATION BY PARTS:**

$$\int f(x)g'(x)\,dx \;=\; f(x)g(x) \;-\; \int f'(x)g(x)\,dx$$

(ii) **SPRING FORCE:**

$$F(x) \;=\; kx$$

(iii) **HYDROLIC FORCE:**

$$F(h) \;=\; \rho g h A(h)$$

(iv) **CENTER OF MASS:**

$$\bar{x} \;=\; \frac{\int_a^b x f(x)\,dx}{\int_a^b f(x)\,dx} \;=\; \frac{\int_a^b x f(x)\,dx}{A}$$

$$\bar{y} \;=\; \frac{\int_a^b \frac{1}{2}[f(x)]^2\,dx}{\int_a^b f(x)\,dx} \;=\; \frac{\int_a^b \frac{1}{2}[f(x)]^2\,dx}{A}$$

BASIC INTEGRALS:

(1)
$$\int u^n \, du = \frac{u^{n+1}}{n+1} + c \quad , \quad n \neq -1$$

(2)
$$\int \frac{1}{u} \, du = Ln(u) + c$$

(3)
$$\int \frac{1}{u^2+1} \, du = tan^{-1}(u) + c$$

(4)
$$\int sin(u) \, du = -cos(u) + c$$

(5)
$$\int cos(u) \, du = sin(u) + c$$

(6)
$$\int sec^2(u) \, du = tan(u) + c$$

(7)
$$\int csc^2(u) \, du = -cot(u) + c$$

(8)
$$\int sec(u)tan(u) \, du = sec(u) + c$$

(9)
$$\int csc(u)cot(u) \, du = -csc(u) + c$$

ROCHESTER INSTITUTE OF TECHNOLOGY
DEPARTMENT OF MATHEMATICS & STATISTICS

COMPLEX VARIABLES 1016 – 420 – 02

TEST # 3

NOVEMBER 4TH , 2004

Name:_____

There are **6 questions** on the test. Each question will be **20 points each**.

YOU MUST SHOW ALL WORK on each problem. You will not receive any credit for any problem done correctly without proper work. In particular, you **MUST SHOW**:

(i) Explain all the method(s) used to evaluate each integral.
(ii) Draw diagram(s) of curves in Line Integral(s).
(iii) Show all the necessary steps of substitutions; in integrals and in series as well.
(iv) Explain why series converge or diverge.

There are **ABSOLUTELY NO CALCULATORS** allowed on the test.

Good luck. Do not rush through the test.

(1) Determine the following **INTEGRALS**; (10 points each):

(a)

$$\int_{|z+4i|=2} \frac{z^2}{z^4-1}\, dz\ .$$

MUST SKETCH THE DIAGRAM AND EXPLAIN WHAT METHOD(S) YOU ARE USING.

PART (B) IS ON THE NEXT PAGE

(b)

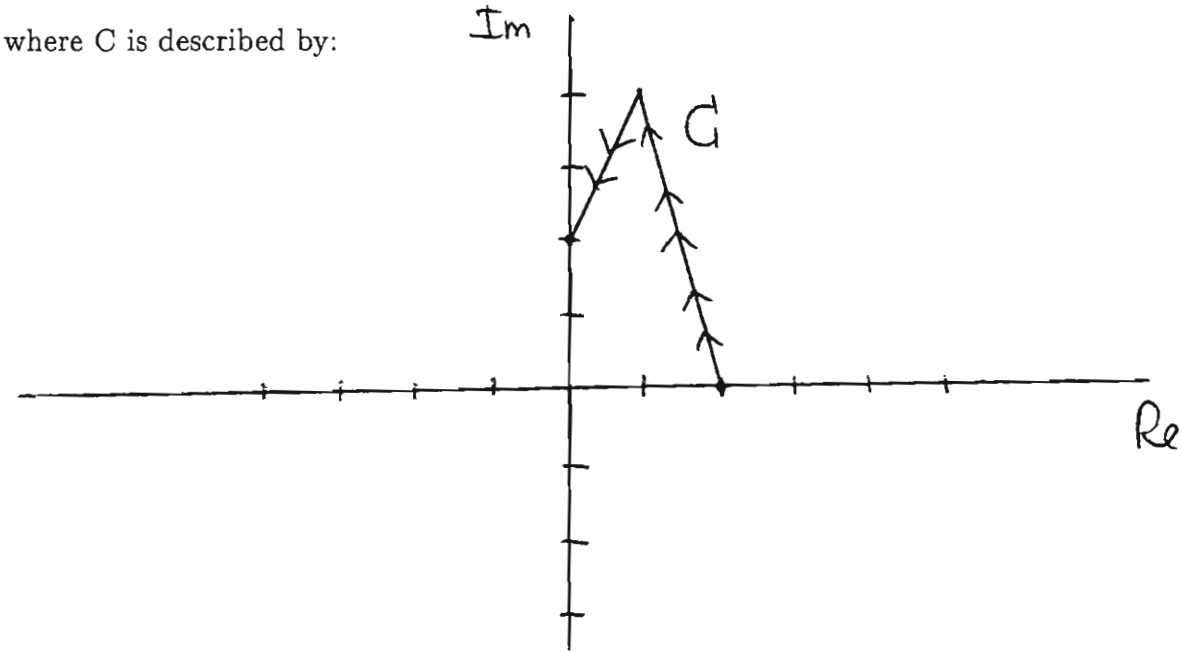
$$\int_{|z|=1} \frac{1}{z}\, dz\ .$$

MUST SKETCH THE DIAGRAM AND EXPLAIN WHAT METHOD(S) YOU ARE USING.

(2) Determine the following **INTEGRAL**; (20 points):

$$\int_C z^3\, dz\ .$$

where C is described by:



MUST SKETCH THE DIAGRAM AND EXPLAIN WHAT METHOD(S) YOU ARE USING.

CONTINUE ON THE NEXT PAGE IF NECESSARY

(3) Determine the following **INTEGRAL**; (20 points):

$$\int_{|z-2i|=1} \frac{z}{z^2+4}\, dz\ .$$

MUST SKETCH THE DIAGRAM AND EXPLAIN WHAT METHOD(S) YOU ARE USING.

CONTINUE ON THE NEXT PAGE IF NECESSARY

ATTACHED SHEET FOR NUMBER 3

(4) Determine the following **INTEGRAL**; (20 points):

$$\int_{|z-2i|=2.5} \frac{z}{(z+i)z^4}\, dz\ .$$

MUST SKETCH THE DIAGRAM AND EXPLAIN WHAT METHOD(S) YOU ARE USING.

CONTINUE ON THE NEXT PAGE IF NECESSARY

ATTACHED SHEET FOR NUMBER 4

(5) Determine the following properties of the given series:

(a) If the following series **CONVERGE** (7 points):

$$\sum_{n=0}^{\infty} \left(\frac{2i}{2+i}\right)^n$$

MUST EXPLAIN YOUR REASONING WHY

(b) If the series in part (a) converge, determine their **SUM** (5 points).

PART (C) IS ON THE NEXT PAGE

(c) The **RADIUS** and the **REGION OF CONVERGENCE** of the following series (8 points):

$$\sum_{n=1}^{\infty} \frac{(z+3i)^n}{n^2}$$

(6) Write the **TAYLOR'S EXPANSION** of the following functions:

(a) (10 points)

$$f(z) = e^{\frac{1}{z}}$$

PART (B) IS ON THE NEXT PAGE

ROCHESTER INSTITUTE OF TECHNOLOGY
DEPARTMENT OF MATHEMATICS & STATISTICS

COMPLEX VARIABLES 1016 – 720 – 02

TEST # 3

NOVEMBER 4TH , 2004

Name:_____

There are **6 questions** on the test. Each question will be **30 points each**.

YOU MUST SHOW ALL WORK on each problem. You will not receive any credit for any problem done correctly without proper work. In particular, you **MUST SHOW**:

(i) Explain all the method(s) used to evaluate each integral.
(ii) Draw diagram(s) of curves in Line Integral(s).
(iii) Show all the necessary steps of substitutions; in integrals and in series as well.
(iv) Explain why series converge or diverge.
(v) Show all the steps in all the proofs

There are **ABSOLUTELY NO CALCULATORS** allowed on the test.

Good luck. Do not rush through the test.

(1) Determine the following **INTEGRALS**; (15 points each):

(a)

$$\int_{|z|=3} \frac{z^2}{z^2+4}\, dz\ .$$

MUST SKETCH THE DIAGRAM AND EXPLAIN WHAT METHOD(S) YOU ARE USING.

CONTINUE ON THE NEXT PAGE IF NECESSARY

ATTACHED SHEET FOR PART a

PART (B) IS ON THE NEXT PAGE

(b)

$$\int_{|z-2|=1.5} \frac{1}{z^4 - 1}\, dz\ .$$

MUST SKETCH THE DIAGRAM AND EXPLAIN WHAT METHOD(S) YOU ARE USING.

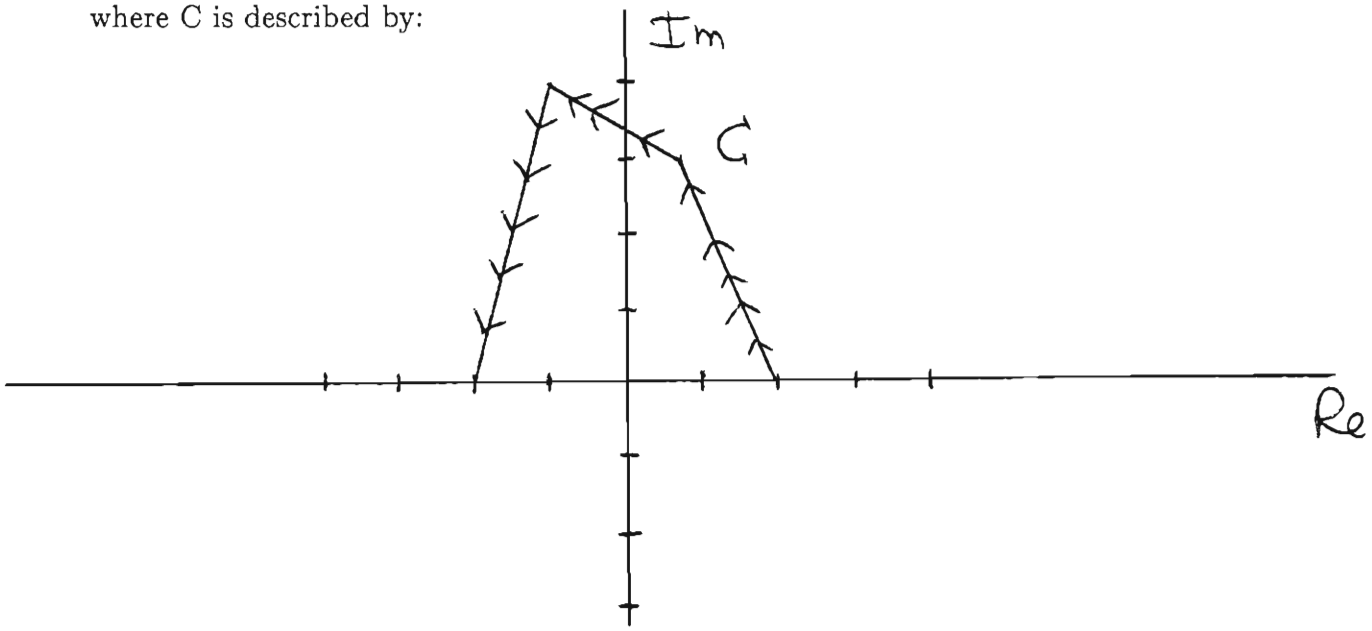
CONTINUE ON THE NEXT PAGE IF NECESSARY

ATTACHED SHEET FOR PART b

(2) Determine the following **INTEGRAL**; (30 points):

$$\int_C z^4 \, dz \, .$$

where C is described by:



MUST SKETCH THE DIAGRAM AND EXPLAIN WHAT METHOD(S) YOU ARE USING.

CONTINUE ON THE NEXT PAGE IF NECESSARY

ATTACHED SHEET FOR NUMBER 2

(3) State and Prove the **ANNULUS THEOREM**; (30 points):

MUST SKETCH THE DIAGRAM(S) AND EXPLAIN WHAT METHOD(S) YOU ARE USING.

CONTINUE ON THE NEXT PAGE IF NECESSARY

ATTACHED SHEET FOR NUMBER 3

(4) Suppose that f(z) is analytic on the ANNULUS

$$0 < |z - z_0| \leq r$$

PROVE that; (30 points):

$$f(z_0) = \frac{1}{2\pi} \int_0^{2\pi} f(z_0 + re^{it})\, dt \ .$$

MUST SKETCH THE DIAGRAM AND EXPLAIN WHAT METHOD(S) YOU ARE USING.

CONTINUE ON THE NEXT PAGE IF NECESSARY

ATTACHED SHEET FOR NUMBER 4

(5) Determine the following properties of the given series:

(a) If the following series:

$$\sum_{n=1}^{\infty} \frac{i^n}{n^2}$$

Converge Conditionally, **Converge Absolutely**, or **Diverge**; (10 points).

MUST EXPLAIN YOUR REASONING WHY

PART (B) IS ON THE NEXT PAGE

(b) The **RADIUS** and the **REGION OF CONVERGENCE** of the following series; (10 points):

$$\sum_{n=1}^{\infty} \frac{(z+3i)^n}{n^2}$$

PART (C) IS ON THE NEXT PAGE

(c) **PROVE** the following; (10 points):

$$\sum_{n=N}^{\infty} c_n \quad \text{converges}$$

for all $N = 0, 1, 2, \ldots$ implies that

$$\sum_{n=0}^{\infty} c_n \quad \text{converges}$$

(6)

(a) By using the TAYLOR'S SERIES, **PROVE** the following property; (20 points):

$$e^{it} = cos(t) + i\, sin(t)$$

PART (B) IS ON THE NEXT PAGE

(b) Write the **TAYLOR'S EXPANSION** of the following function; (10 points):

$$f(z) = \frac{z^2}{1 - z^4}$$

ROCHESTER INSTITUTE OF TECHNOLOGY
DEPARTMENT OF MATHEMATICS & STATISTICS

COMPLEX VARIABLES 1016 – 420 – 02

FINAL EXAM

NOVEMBER 15th , 2004

Name:_____

There are **16 QUESTIONS** on the exam. You **MUST DO** the following two questions:

- QUESTION # 1
- QUESTION # 2

Then **CHOOSE** any other **9 QUESTIONS**.
You must do **11 QUESTIONS** all together.
Each question will be **14 POINTS EACH**.

YOU MUST SHOW ALL WORK on each problem. You will not receive any credit for any problem done correctly without proper work.

NOTE: You **MUST** state the Method that you are using to solve each problem.

YOU MUST **INDICATE CLEARLY** WHICH QUESTION YOU WANT ME TO EVALUATE.

PROBLEM CHECK LIST:

INDICATE which question you are doing. **PUT A CHECK** next to the question which you choose to do:

QUESTION # 1 _____ **THIS PROBLEM MUST BE DONE**

QUESTION # 2 _____ **THIS PROBLEM MUST BE DONE**

QUESTION # 3 _____

QUESTION # 4 _____

QUESTION # 5 _____

QUESTION # 6 _____

QUESTION # 7 _____

QUESTION # 8 _____

QUESTION # 9 _____

QUESTION # 10 _____

QUESTION # 11 _____

QUESTION # 12 _____

QUESTION # 13 _____

QUESTION # 14 _____

QUESTION # 15 _____

QUESTION # 16 _____

* THIS PROBLEM MUST BE DONE *

(1) Consider the function:

$$f(z) = \frac{Log(z)}{z - 2i} \quad \text{on the curve} \quad |z - 3i| = 4$$

Determine; (14 points):

$$Res[f(z), 2i]$$

CONTINUE ON THE NEXT PAGE IF NECESSARY

ATTACHED SHEET FOR NUMBER 1

* THIS PROBLEM MUST BE DONE *

(2) **REWRITE** the following Integral as a **COUNTOUR INTEGRAL**; (14 points):

$$\int_0^{2\pi} \frac{sin^2(t)}{4 + cos(t)} dt \ .$$

MUST SHOW ALL THE SUBSTITUTIONS.

DO NOT EVALUATE THE INTEGRAL.

CONTINUE ON THE NEXT PAGE IF NECESSARY

ATTACHED SHEET FOR NUMBER 2

(3) Let

$$z = a + bi \qquad \text{and} \qquad w = c + di \ .$$

Prove the following property (14 points):

$$\overline{\left[\frac{z}{w}\right]} = \frac{\overline{z}}{\overline{w}}$$

(4) We define the **COMPLEX EXPONENTIAL** as:

$$e^{i\theta} =$$

Using the above definition, prove the following property (14 points):

$$\frac{1}{e^{i\theta}} = e^{-i\theta} = \overline{e^{i\theta}}$$

Furthermore, use the following three identities to prove the properties:

(i)
$$sin^2(\theta) + cos^2(\theta) = 1.$$

(ii)
$$cos(-\theta) = cos(\theta).$$

(iii)
$$sin(-\theta) = -sin(\theta).$$

(5) Determine the following expression in **RECTANGULAR FORM**; (14 points):

$$\sqrt[4]{-1 + \sqrt{3}i}$$

(6) Solve the following equation (14 points):

$$z^6 = 1$$

NOTE: You may leave your answers in **POLAR FORM**.

CONTINUE ON THE NEXT PAGE IF NECESSARY

ATTACHED SHEET FOR NUMBER 6

(7) Consider the following function:

$$f(z) = \overline{z} + \frac{1}{z}$$

(a) Rewrite the function in the form; (7 points):

$$u(x, y) + iv(x, y)$$

(c) Using part(a), rewrite the function in the form; (7 points):

$$u(r, \theta) + iv(r, \theta)$$

(8) Consider the function in the form

$$f(z) = u(x,y) + iv(x,y)$$

We say that f(z) is **DIFFERENTIABLE** if it satisfies the following two **CAUCHY RIEMANN EQUATIONS**:

Determine if the following function is **DIFFERENTIABLE**; (14 points):

$$f(z) = e^z$$

CONTINUE ON THE NEXT PAGE IF NECESSARY

ATTACHED SHEET FOR NUMBER 8

(9) Consider a **COMPLEX EXPONENTIAL FUNCTION**:

$$f(z) = e^z$$

Let $z = x + iy$.
PROVE the following property (14 points):

$$\left|e^{i\overline{z}}\right| = e^y$$

HINT: Use the identity

$$sin^2(\theta) + cos^2(\theta) = 1.$$

(10) We define a **COMPLEX LOGARITHMIC FUNCTION** as:

$Log(z) \; =$

PROVE the following property (14 points):

$$Log\left(\frac{z}{w}\right) \; = \; Log(z) \; - \; Log(w)$$

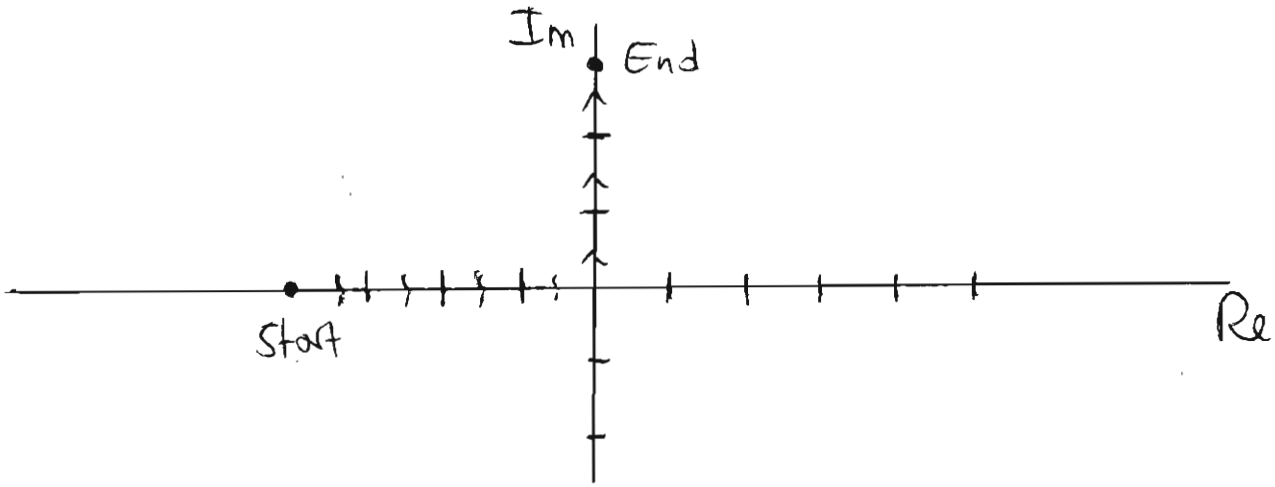
(11) By using the identity(ies):

$$\begin{aligned} sin(\alpha+\beta) &= sin(\alpha)cos(\beta) + sin(\beta)cos(\alpha) \\ sin(\alpha-\beta) &= sin(\alpha)cos(\beta) - sin(\beta)cos(\alpha) \end{aligned}$$

PROVE the following property (14 points):

$$\overline{sin(z)} = sin(\overline{z})$$

(12) Consider a curve C described by:



WRITE the following **Definite Line Integral(s)** (14 points).

NOTE: **WRITE** the definite integral(s) only. **DO NOT EVALUATE.**

$$\int_C (\overline{z})^2 \, dz \, .$$

CONTINUE ON THE NEXT PAGE IF NECESSARY

ATTACHED SHEET FOR NUMBER 12

(13) Determine the following **INTEGRAL**; (14 points):

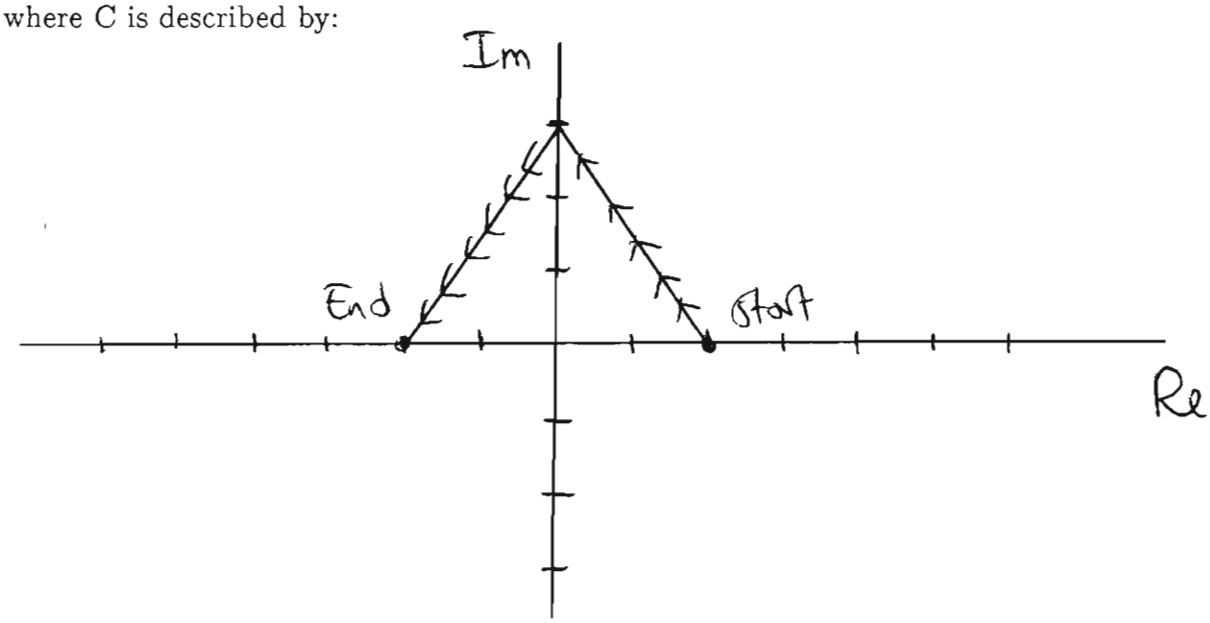
$$\int_{|z-4i|=2} \frac{z^2}{z^4-1}\, dz\ .$$

MUST SKETCH THE DIAGRAM AND EXPLAIN WHAT METHOD(S) YOU ARE USING.

(14) Determine the following **INTEGRAL**; (14 points):

$$\int_C z^2 \, dz \, .$$

where C is described by:



MUST SKETCH THE DIAGRAM AND EXPLAIN WHAT METHOD(S) YOU ARE USING.

CONTINUE ON THE NEXT PAGE IF NECESSARY

ATTACHED SHEET FOR NUMBER 14

(15) Determine the following **INTEGRAL**; (14 points):

$$\int_{|z+2i|=2.5} \frac{1}{(z-i)z^5}\, dz\ .$$

MUST SKETCH THE DIAGRAM AND EXPLAIN WHAT METHOD(S) YOU ARE USING.

CONTINUE ON THE NEXT PAGE IF NECESSARY

ATTACHED SHEET FOR NUMBER 15

(16)

(a) Determine if the following series **CONVERGE** (7 points):

$$\sum_{n=0}^{\infty} \frac{(1-i)^{4n}}{n!}$$

MUST EXPLAIN YOUR REASONING WHY

PART (B) IS ON THE NEXT PAGE

(b) The **RADIUS** and the **REGION OF CONVERGENCE** of the following series (7 points):

$$\sum_{n=1}^{\infty} \frac{(z+3i)^n}{n^2}$$