Use of decision science in mechanical engineering design

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ABSTRACT

The integration of decision science into design techniques in Engineering and Engineering Technology programs is necessary to provide graduating engineers the necessary skills to become more immediate contributors to the goals and profits of their chosen companies. Example teaching and analysis techniques are discussed, which will allow faculty to introduce and reinforce effective design into Mechanical Engineering courses. These techniques can be applied to other courses also.

INDEX TERMS

Cost-effective design, Decision matrix, Decision science, Engineering materials, Mechanics, Pedagogy, Strength of materials

I. INTRODUCTION AND BACKGROUND

In over 30 years of work with engineers, designers, and architects, it has been observed many have difficulty determining the proper combination of material and shape to meet design and cost criteria. There are a number of recognized methods available to evaluate the structural rigidity or integrity of design components. However, many design professionals lack the ability to incorporate decision analysis and cost effectiveness into their design. How do you get the most rigidity for the least cost and, in many cases, at the lowest weight? That is to say, "the most bang for your buck." Graduating Engineering and Engineering Technology students do not have a good grasp of this concept, and it is suggested that faculty have the responsibility to introduce and nurture decision analysis in design. It is the purpose of this paper to demonstrate one method of introducing this concept to Mechanical Engineering students in typical Strength of Materials courses. Rigidity will be defined considering both the material and the shape of the cross section. Different combinations of material and shape will be evaluated. A simple decision matrix will be shown as one method of comparison, and the entire concept will be pulled together. This concept should be incorporated into a variety of other Engineering and Engineering Technology courses to demonstrate and reinforce its application.

II. PROBLEM STATEMENT

A. Definitions and Scope

The *rigidity* (or stiffness) of a material is simply a measure of the amount of deflection, δ, that occurs when a simple cantilevered beam is exposed to some applied load as shown in Figure 1.

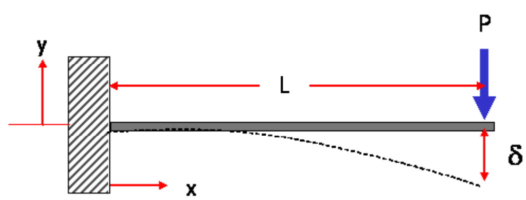

Figure 1. A simple cantilevered beam showing an applied load at the end of the beam and depicting the amount of deflection [1]

The amount of deflection, δ, is a function of *both* a material property and the cross-sectional shape of the beam. The *material property* is the Modulus of Elasticity, E, of the material being used and can be determined by a simple tension test or found in published literature. Normally, the Modulus of Elasticity is a constant for each specific metal, but can vary by molecular weight in polymers. The *shape property* is the Moment of Inertia, I, of the cross-sectional shape, which can be determined using a number of mathematical and graphical methods or found in published literature.

The *Modulus of Elasticity* is simply the slope of the elastic portion of the stress-strain curve from a tension test as shown in Figure 2.

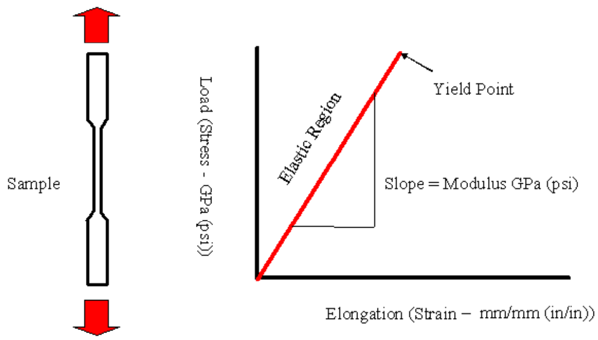

Figure 2. Determination of the modulus of elasticity from a simple tension test

The modulus of various materials is different, but is normally constant for alloys of the same material. Examples are shown in Figure 3.

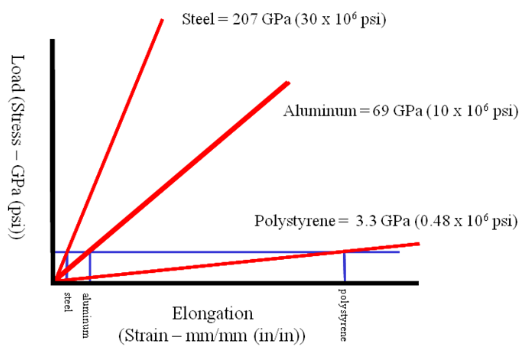

Figure 3. A comparison of the slope of the elastic portion of the stress-strain curve for steel, aluminum, and polystyrene

Intuitively, looking at the slopes in Figure 3, it can be seen that for a given stress level each material will deflect differently simply based on the Modulus of Elasticity, which is a material property. In looking at the deflection of cantilevered beams made from the three different materials, the following can be observed.

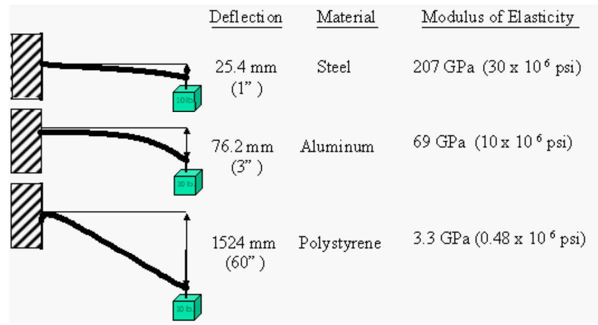

Figure 4. A comparison of modulus for materials with the deflection of a simple cantilevered beam assuming all pieces are equivalent size and shape and neglecting the weight of each member [2], [9]

III. METHODOLOGY

A. Integration of Shape Effect

The material rigidity and density can be compared using the specific stiffness ratio [2], which is the ratio of the Modulus of Elasticity to the density.

$$\text{Specific Stiffness} = S_p = E/\rho$$

For example, in comparing steel to aluminum, the following is observed [3], [8]:

Steel [6] = 207 GPa (30 x 10^6 psi) /
7750 kg/m^3 (0.28 lbs/in^3)
Aluminum [7] = 69 GPa (10 x 10^6 psi) /
2768 kg/m^3 (0.10 lbs/in^3)

This gives an equivalent ratio for each material. Thus, both steel and aluminum are very similar if looking only at the amount of stiffness per pound of weight, not considering the shape of the cross section. The ratio for polystyrene is significantly lower, demonstrating that polymers are much less rigid than most metals.

As will be discussed, the shape effect must always be considered and can be expressed as the Moment of Inertia, I [1]. This is the capacity of a cross section to resist bending. It is always considered with respect to a reference axis such x-x or y-y. It is a mathematical property of a section concerned with the cross-sectional area and how that area is distributed about the reference axis. This reference axis is usually a centroidal axis. This Moment of Inertia is an important value, which is used to determine the state of stress in a section, to calculate the resistance to buckling, and to determine the amount of deflection of a member.

The following is an example to consider. Consider the two 25.4 mm x 101.6 mm (1"x 4") solid bars shown in Figure 5 and determine which will deflect more and why.

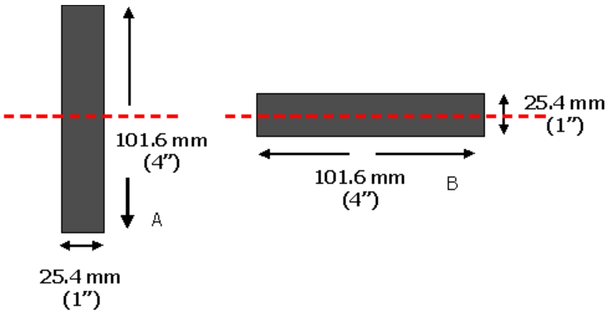

Figure 5. Example depicting the variation of the moment of inertia of the same cross-section oriented relative to the horizontal axis

Bar A has its 25.4 mm (1") dimension parallel to the horizontal axis, while bar B has its 101.6 mm (4") dimension parallel to the horizontal axis. The Moment of Inertia for a rectangular cross-section in relation to the horizontal centroidal axis can be calculated using the following equation [1], [4]:

$$I_x = \frac{bh^3}{12}$$

(1) Moment of Inertia for a rectangular cross section

In (1), b is the length of the base and h is the height of the cross section. Other shaped cross sections require different equations to calculate their moments.

Using (1) and substituting values for the respective base and height dimensions, it is seen bar A has a moment value of 2.21×10^6 mm^4 (5.33 in^4), while bar B has a value of 1.37×10^5 mm^4 (0.33 in^4). Both bars are the same size and shape; however, they are oriented differently. Bar A is significantly more rigid (16 times!) than bar B. Although the cross- sectional area of both bars is the same, it is distributed differently above and below the horizontal axis which results in a greater stiffness for bar A. Intuitively, envision a 50.8 mm x 203.2 mm (2" x 8") piece of dimension lumber. It is clear its rigidity, when oriented with the 50.8 mm (2") dimension oriented parallel to the horizontal axis (like a floor joist), is significantly greater than with 8" dimension parallel to the horizontal axis.

B. Combination of Modulus and Shape

Combining the material property, E, and the shape property, I, into one equation gives the total deflection, δ, of the cantilevered beam as shown in Figure 6 and (2).

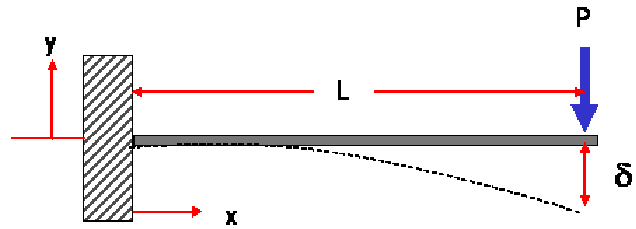

Figure 6. A cantilevered beam used in the determination of the deflection of the end relative to the applied load [1], [4]

$$\delta = \frac{-PL^3}{3EI}$$

δ = Deflection at the end
P = Load
L = Length
E = Modulus of Elasticity (material property)
I = Moment of Inertia (shape factor)

(2) Equation to predict the deflection of the end of a cantilevered beam related to the modulus of elasticity, the moment of inertia, and the applied load [1], [4]

δ is the total deflection for the cantilevered structure as shown in Figure 6. Look again at the 25.4 mm x 101.6 mm (1" x 4") steel, aluminum, and polystyrene bars to see the total deflection of each can be calculated. Assume the applied load, P, is 226.8 kg (500 lbs), the length of the cantilevered bar is 914.4 mm (36"), and the bar has the 25.4 mm (1") dimension parallel to the horizontal axis. Substituting these values into the equation above gives the following results for each beam:

δ_{steel} = - (226.8 kg)(914.4 mm)^3 / 3(207 GPa) (2.25×10^6 mm^4) = -1.241 mm
- (500 lbs)(36 in)^3 / 3(30 x 10^6 psi)(5.3 in^4)
= - 0.0489 in

$\delta_{aluminum}$ = - (226.8 kg)(914.4 mm)^3 / 3(69 GPa)

$(2.25 \times 10^6\ mm^4) = -3.726$ mm
$- (500\ lbs)(36\ in)^3 / 3(10 \times 10^6\ psi)(5.3\ in^4)$
$= - 0.1467$ in

$\delta_{polystyrene} = - (226.8\ kg)(914.4\ mm)^3 / 3(3.3\ GPa)$
$(2.25 \times 10^6\ mm^4) = -77.846$ mm
$- (500\ lbs)(36\ in)^3 / 3(0.48 \times 10^6\ psi)(5.3\ in^4)$
$= - 3.0649$ in

This reveals the aluminum bar deflects three times as much as the steel bar of the same shape. This also demonstrates polystyrene has a huge deflection (60 times greater!) and is probably not a consideration in most designs.

Consider only the steel and aluminum bars. How can the deflection of the aluminum bar be made the same as, or similar to, the steel bar? The answer can be determined by rearranging (2) and solving for I to obtain (3) below.

$$I = - PL^3 / \delta(3)(E)$$

(3) Rearrangement of (2)

Substitute in the value of E for aluminum, 69 GPa (10×10^6 psi). Use a load of 226.8 kg (500 lbs) and a length of 914.4 mm (36") and set the deflection, δ, of the aluminum bar to be -1.241 mm (- 0.0489 in). The result is $6.49 \times 10^6 mm^4$ ($15.61\ in^4$).

Thus, to get a deflection of the aluminum bar equal to the deflection of the steel bar -1.241 mm (- 0.0489 in), an aluminum bar must have a Moment of Inertia, I, equal to $6.49 \times 10^6\ mm^4$ ($15.61\ in^4$). Look at the various cross-sectional shapes available for aluminum and determine which shape has a Moment of Inertia equal to or greater than $6.49 \times 10^6\ mm^4$ ($15.61\ in^4$). One example of a shape that meets this criteria is a 101.6 mm x 152.4 mm (4" x 6") aluminum I-beam, which has an I value of $9.15 \times 10^6\ mm^4$ ($21.99\ in^4$). This gives a total deflection of -0.899 mm (- 0.0354 in), which is 0.342 mm (0.0135 in) less than the steel bar.

Using this method, equation, and E and I values, we can also look at other combinations of material and cross-sectional shape to arrive at the lowest defection characteristics for the lowest density.

In the above comparison, the 25.4 mm x 101.6 mm x 914.4 mm (1"x 4"x 36") steel bar would weigh 18.29 kg (40.32 lbs), and the 101.6 mm x 152.4 mm x 914.4 mm (4"x 6"x 36") aluminum I-beam would weigh only 5.48 kg (12.09 lbs). Thus, an aluminum I-beam has greater rigidity than the steel bar and weighs 12.81 kg (28.23 lbs) less! This evaluation practice is very common in the aerospace and transportation industries, but can be used in just about any situation. Obviously, for beams and parts supported in other ways the deflection equations will be different and can be found in any Strength of Materials text or reference book.

C. Integration of Cost Factor

Cost will be the next consideration. This must be introduced in almost all comparative design processes. This is the part missed in many typical courses. There are many methods that can be used to evaluate the cost factor. One simple process is to look at the material cost per pound in the previous example. This gives the following:

The cost of bulk steel is approximately \$0.55 / kg (\$0.25 /lb) and bulk aluminum is approximately \$2.20/kg (\$1.00 /lb).

25.4 mm x 101.6 mm x 914.4 mm x 77.5×10^{-6} kg/mm^3 = 18.29 kg
(1"x 4" x 36" x 0.28 lbs $/in^3$ = 40.32 lbs)
\$0.55/kg x 18.29 kg = \$10.08 for the steel bar
(\$0.25 / lb x 40.32 lbs = \$10.08 for the steel bar)

101.6 mm x 152.4 mm x 914.4 mm aluminum I-beam weighs 5.997×10^{-3}kg/mm = 5.49 kg
(4" x 6" x 36" aluminum I-beam weighs 4.03 lbs / ft = 12.09 lbs)
\$2.20 / kg x 5.49 kg = \$12.09 for the aluminum I-beam
(\$1.00 / lb x 12.09 lbs = \$12.09 for the aluminum I-beam)

Therefore, the aluminum I-beam gives less deflection and costs only \$2.01 more than the steel bar based on bulk prices.

Another process is to consider the actual costs per foot of the above bar and I-beam. Quoted price for 25.4 mm x 101.6 mm (1"x 4") 1020 cold-rolled-steel bar is \$16.60 per foot, and the 101.6 mm x 152.4 mm (4"x 6") aluminum I-beam is \$16.14 per foot [10]. The cost for 914.4 mm (36") sections of each is \$49.80 for the steel bar and \$48.42 for the aluminum I-beam. So on an actual cost basis, the aluminum I-beam is less expensive, significantly lighter weight, and more rigid. This demonstrates an example of cost-effective design. In an aircraft, automobile, or boat, this weight difference is significant because the weight factor is one of the most important Design Criteria. Ultimately, cost is almost always the major consideration in the real world and should be well understood by students.

IV. RESULTS AND ANALYSIS

A. Decision Matrix

To evaluate the above factors, a simple selection model such as a decision matrix (Table II) can be used. To create a decision matrix, follow these steps:

- Establish the Design Criteria. In our example, the Design Criteria might include deflection, weight, cost, size, and safety. Many other criteria, such as manufacturing cost, availability, durability, and fatigue resistance, can also be included at the evaluator's discretion.
- Assign a weighting factor to each of the Design Criteria based upon the relative importance of each. This weighting factor can be arrived at using numerous methods, including design factor, history, failure analysis, environment, experience, committee consensus, etc. These are subjective ratings and may be tailored to any situation. Historical experience is one of the more common methods in establishing weight factors. If there are disagreements among evaluators, the importance can be further evaluated using a sensitivity analysis. Since there are five Design Criteria in this example, a five point scale (1-5) could be used. A weighting factor of 1 would be the least important and 5 would be the most important.
- Develop a list of Design Alternatives or, in our case, material and shape combination options: 25.4 mm x 101.6 mm (1" x 4") steel bar, 25.4 mm x 101.6 mm (1" x 4") aluminum bar, 25.4 mm x 101.6 mm (1" x 4") polystyrene bar, and 101.6 mm x 152.4 mm (4" x 6") aluminum I-beam.

Establish a Rating Factor, which indicates the performance of the Design Alternative with respect to each Design Criteria. These could be as follows:

Table I. Rating factors

RATING FACTOR	DEFINITION
1	Failure
2	Low Performance
3	Average Performance
4	High Performance
5	Outstanding Performance

- Use the above five Rating Factors to rate each Design Criteria for each of the Design Alternatives.
- Multiply each Rating Factor by each of the weighting factors and obtain a value for each Design Alternative.
- Finally, the best design alternative is determined by summing the respective Value
- Columns within the decision matrix. The column with the highest sum is the best choice.

Table II. Decision matrix [5]

		25.4 x 101.6 mm (1" x 4") Steel Bar		25.4 x 101.6 mm (1" x 4") Aluminum Bar		25. x 101.6 mm (1" x 4") Polystyrene Bar		101.6 x 152.4 mm (2" x 6") Aluminum I-beam	
Design Criteria	Weight Factor	Rating	Value	Rating	Value	Rating	Value	Rating	Value
Deflection	**5**	5	25	4	20	1	5	5	25
Weight	**4**	1	4	5	8	5	20	5	20
Cost	**5**	4	20	3	25	2	10	5	25
Size	**2**	4	8	4	8	3	6	3	8
Safety	**3**	4	12	4	12	1	3	4	12
Totals			**69**		**73**		**44**		**90**

Clearly, the aluminum I-beam is the best option. The aluminum bar has the second highest total, so it might be an alternative if the deflection meets the design standard.

V. CONCLUSIONS

To summarize, we as educators have the responsibility to teach Engineering and Engineering Technology students all aspects of design. Based on my years of experience in industrial design and manufacturing, we fall short of the goal in many courses because we do not introduce or emphasize the economics and design alternative factors. Many students graduate, begin a job, and are shocked that the design they must create for their company is not always the best mechanical design. In most cases, companies are looking for an sufficient design that is the lowest cost or the lightest weight. In many situations, it is possible [to satisfy both conditions (the low cost and lightest weight)] as shown in the example above.

VI. RECOMMENDATIONS

These are just a few examples of how these factors can be introduced in typical mechanical design courses. There are other ways to give students the complete picture, and I encourage you to seek them out and ensure your students understand these concepts. They will be more effective engineers and will be immediate contributors to their company of choice.

As Program Coordinator, I have begun to integrate this type of analysis into other courses such as Statics, Machine Design, Computer Aided Design, and Analysis of Mechanisms. The overall plan is to use this as a thread to tie problems, analysis, and programs together.

VII. REFERENCES

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