Design fatigue curves based on small crack growth and crack closure

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ABSTRACT

The propagation behavior of short cracks cannot be studied by linear elastic methods because of the large plastic region near the crack tip, as well as a break down in correlation of the stress intensity factor with the fatigue crack growth rates. The proposed fatigue design approach incorporates a distance parameter in conjunction with linear elastic fracture mechanics and effectively integrates long and short crack growth test data. This distance parameter is a material constant, which allows for the effects of (a) large-scale plasticity, (b) crack closure, and (c) fatigue crack threshold. Furthermore, this parameter successfully predicts fatigue crack growth behavior of short cracks. The practical application of this method is for studying fatigue crack initiation in pressure vessels and is based on the concept that initiation occurs only when the material ahead of the crack tip is damaged enough by cyclic straining. In this paper, the initiation and growth of small cracks have been investigated along with consideration for crack closure. These results provide the design fatigue curves for some typical structural materials and lead to realistic estimates of fatigue lives for materials used in pressure vessels. The techniques outlined in this paper are equally applicable to materials used in aerospace and automotive industries.

INDEX TERMS

Crack closure, Distance parameter, Fatigue curves, Small cracks.

I. INTRODUCTION

The theories of fatigue initiation based on critical distance (or process zone) concepts have been used for many years. Peterson [14] first reported the size effect when he noticed that the mean fatigue life and variation in fatigue life were a function of the stressed volume. The size effect plays a key role in controlling high cycle fatigue because damage accumulation often starts on a small scale. High cycle fatigue failures are not usually initiated by the large microstructural defects associated with low cycle fatigue failures, but often nucleate "naturally" at local regions of high stress. The damage progresses through mechanisms starting with crack nucleation, microstructurally small crack growth, followed by linear elastic long crack growth. Each mechanism is associated with a characteristic size, and each characteristic size has its own geometric complexity, constitutive law, and heterogeneity. Fatigue behavior cannot be fully understood and predicted without obtaining information about each of the characteristic sizes. According to the observations of Peterson [14] and of Kitagawa and Takahashi [7], fatigue crack growth behavior of short cracks differs in a non-conservative manner from expectations based upon long crack behavior.

There are three characteristics responsible for this anomaly:

a. Plastic zone size at the crack tip of a short crack is large with respect to the length of the crack; thus, violating the requirement of linear elastic fracture mechanics.
b. Overall applied stress levels may be high with respect to the yield strength of the material; thus, violating the small scale yielding appropriate for linear elastic fracture mechanics.
c. Level of crack closure in a state of transition as the crack changes from a short crack to a long crack.

Since crack closure reduces the effective range of the stress intensity factor, there will be a greater driving force for a short crack as compared to the long crack for a given stress intensity factor range. Hence, the crack growth rate for a short crack will be higher than for a short crack.

Kitawaga and Takahashi [7] showed that below a critical crack size, the threshold stress intensity factor range, ΔK_{th},, for small cracks decreases with decreasing crack lengths; and the threshold stress range, $\Delta\sigma_{th}$,, approaches the smooth bar endurance stress range, $\Delta\sigma_e$, at very small crack lengths. Experimental studies show the threshold associated with long and small cracks can be very different. The long crack threshold, ΔK_{th}, for a particular material is independent of crack length. Therefore, the threshold condition for long cracks is one of constant stress intensity factor range, $\Delta K_0 = \Delta Kth$ and for small cracks is of constant stress range, namely the endurance stress range, $\Delta\sigma_e$.

II. BACKGROUND

Chan [1] has developed a fatigue crack initiation model based on microstructure, which includes explicit crack size and microstructure scale parameters. Hanlon, Kwon, and Suresh [6] have studied the fatigue response of electrodeposited pure nickel and an ultra-fine crystalline Al-Mg alloy. They found that grain refinement leads to an increase in resistance to failure under stress-controlled fatigue accompanied by poor resistance to fatigue crack growth. Chu *et al* [2] have studied the cyclic deformation and crack initiation and propagation in α-iron polycrystals with particular reference to the orientation of grains. Krupp [8] in his recent treatise has identified the missing link between the microstructural dimensions of the fatigue process on the atomic scale and the engineering design concepts of structures subject to complex loading.

The stress intensity range required to propagate a crack must remain finite rather than become infinitely large when the length of the short crack is extremely small. The value of the critical distance is determined from experimental results. As such, it is an empirical parameter to be regarded as a material constant. El Haddad, Dowling, Topper, and Smith [4] demonstrated the need for a material constant (distance parameter) in dealing with short crack behavior. This parameter appears in different forms in a number of theories of fatigue crack initiation. Mura and Tanaka [11] propose a theory of fatigue crack initiation dependent among other parameters, such as the grain size; the smaller the grain size, the higher the resistance to crack initiation. Miller [10] obtained a best-fit equation to the micro structural short crack growth data for medium carbon steels in the form of a fatigue crack growth parameter *da/dN* proportional to the stress range raised an exponent times the difference of the grain diameter and the crack length. In this work, the grain diameter is interpreted as the distance parameter "*d*." The upper bound on micro structural short crack growth is considered a constant equal to the grain diameter, "*d*." If the threshold crack length exceeds "*d*," then failure does not occur, but non-propagating cracks may be found. However, if, the threshold crack length is less than "*d*," then failure occurs. In a recent study, Taylor [15] has developed an analytical method for finding the critical distance for a given material and showed the appropriate distance is twice the constant of El Haddad's [4]. Taylor also states the critical distance approach has similar implication as LEFM, in that it is also empirical, which assumes two cracks behave identically if certain features of their stress fields are identical. McEvily, Eifler, and Macherauch [9] report modification to the LEFM approach to establish a new parameter capable of correlating both long crack and short crack growth data. These modifications include the use of a distance parameter (a material constant), along with considerations of large-scale plasticity, crack closure, and fatigue crack threshold. Roche [13] employed a material parameter "*d*" and has developed a practical design rule for crack-like geometrical discontinuities. Roche's [13] approach and the approach indicated by El Haddad et al [4] have been compared and contrasted in this paper.

III. PROBLEM STATEMENT

In this work, fatigue crack initiation has been studied for a typical pressure vessel material using the concept of distance parameter. The endurance limit is characterized by the maximum value of the local stress range, which generally occurs on the surface. For long cracks, fatigue failure is determined by the stress intensity factor range, ΔK. This range is only an approximation to the stress field. For example, given two cracks of different lengths, both located in stress fields of uniform tension, if ΔK is the same for both, then their stress versus distance curves will be identical only at small radius. At large r, each curve will tend to the remote, applied stress value, which will be different for two cracks, and not equal to zero as expected from the definition of K. Therefore, it is not the entire stress field that is important, but only the stress field close to the crack tip.

Traditionally, fatigue crack initiation has been treated by empirically determining the nominal stress amplitude that can be applied to a smooth specimen for an infinite number of cycles. This is the endurance stress range, $\Delta\sigma_e$ (twice the endurance limit, S_e). The application of fracture mechanics to fatigue has resulted in a definition of the threshold stress intensity factor range, ΔK_{th}, below which no detectable crack propagation occurs for cracked members. Measurement of ΔK_{th} establishes a range of crack sizes and stress amplitudes for which no further crack extension takes place.

IV. METHODOLOGY

Following Roche [13], the stress range at a distance "*d*" from the crack tip is denoted as $\Delta\sigma$, a value that is less than the endurance stress range, $\Delta\sigma e$ (twice the endurance limit, S_e). Denoting ΔS as the nominal stress range, for short cracks, an equation can be written as:

$$\Delta\sigma = \frac{\Delta K}{\sqrt{2\pi\, d}} + \Delta S \qquad (1)$$

The criterion for initiation can then be expressed as:

$$\Delta\sigma_e = \frac{\Delta K}{\sqrt{2\pi\, d}} + \Delta S \qquad (2)$$

Denoting ΔK_{th}^{l} as the long crack threshold stress intensity factor range, and recognizing the fact that for long cracks, ΔS approaches zero, equation (2) yields:

$$d = \frac{1}{2\pi}\left(\frac{\Delta K_{th}^{l}}{\Delta\sigma_e}\right)^2 \qquad (3)$$

Denoting the crack length as "*a*", and substituting $\Delta K = \Delta S\sqrt{(\pi a)}$, and the value of "*d*" from equation (3) into equation (2) we obtain the criterion for initiation as:

$$\Delta S = \frac{\Delta\sigma_e}{1+\sqrt{\frac{a}{2d}}} \qquad (4)$$

The corresponding distance parameter used in El Haddad's [4] work is denoted as l_0 and is given by:

$$l_0 = \frac{1}{\pi}\left(\frac{\Delta K_{th}^{l}}{\Delta\sigma_e}\right)^2 \qquad (5)$$

and

$$\Delta S = \Delta\sigma_e\sqrt{\frac{1}{1+\frac{a}{l_0}}} \qquad (6)$$

The number of cycles to failure is determined by adding the two cases, namely:

a. Small crack initiation stage followed by its growth to a long crack (from zero to initiation size, a_{th}.
b. The growth of long crack from the value at initiation, ath to the value corresponding to the distance parameter, *d.*

Number of cycles to initiate, N_i

The initiation size (denoted by a_{th}) for a particular stress range is determined first, and is followed by the determination of the number of cycles (N_i) required to grow the crack from a zero length to this size. This size is determined as:

$$a_{th} = \frac{1}{\pi}\left(\frac{\Delta K_{th}}{\Delta S}\right)^2 \qquad (7)$$

For initiation, the equation provided by Miller [10] is employed. This is valid for medium carbon steels and will be assumed to apply for SA 533 Gr. B:

$$\frac{da}{dN} = 1.475x10^{-41}(\Delta S)^{11.49}(d-a) \qquad (8)$$

Integrating this equation from the crack lengths going from zero to a_{th}.

$$N_i = \frac{1}{1.475x10^{-41}(\Delta S)^{11.49}} \int_0^{a_{th}} \frac{da}{(d-a)} \quad (9)$$
$$= \frac{1}{1.475x10^{-41}(\Delta S)^{11.49}} \ln\left(\frac{d}{d-a_{th}}\right)$$

c. Number of cycles to propagate a long crack from a_{th} to d, N_p

The large crack growth data for A533B are due to Dowling [3], where the crack growth per cycle (da/dN) is provided in terms of the cyclic elastic-plastic J-integral (ΔJ). Using SI units the relationship is:

$$\frac{da}{dN} = 3.0x10^{-3}\,\Delta J^{1.587} \quad (10)$$

Integrating this equation for crack lengths going from ath to d the following expression is obtained for the number of cycles:

$$\Delta J = \frac{(\Delta K)^2}{E} = \frac{\left[\Delta S\sqrt{\pi a}\right]^2}{E} = \frac{\pi(\Delta S)^2}{E}a$$
$$\frac{da}{dN} = 3x10^{-3}\left(\frac{\pi(\Delta S)^2}{E}\right)^{1.587} a^{1.587}$$
$$N_p = [\frac{1}{3x10^{-3}\left(\frac{\pi(\Delta S)^2}{E}\right)^{1.587}}] \int_{a_{th}}^{d} \frac{da}{a^{1.587}} \quad (11)$$

$$N_p = \left[\frac{1}{3x10^{-3}\left(\frac{\pi(\Delta S)^2}{E}\right)^{1.587}}\right]\left[\frac{1}{0.587}\right]\left[a_{th}^{-0.587} - d^{-0.587}\right] \quad (12)$$

The associated calculations are shown in Table III. For the material SA 533 Gr. B, an endurance limit of 500 MPa, a threshold stress intensity factor range ΔK_{th} = 8 MPa√m, a distance parameter d = 0.07 mm (7x10-5m) and a modulus of elasticity, E = 200 GPa were used. The corresponding S-N curve is shown in Figure 4.

d. Crack closure effects

The incorporation of closure effects in terms of the effective stress intensity factor range, ΔK_{eff} involves data on crack opening stress, S_o as well as the maximum stress S_{max} and the minimum stress S_{min}. Elber [5] provides the effective stress intensity factor range as:

$$\Delta K_{eff} = \left(\frac{S_{max} - S_o}{S_{max} - S_{min}}\right)\Delta K \quad (13)$$

For the case of completely reversed stress, Newman et al [12] obtain the crack opening stress ratio S_o/S_{max} in terms of S_{max}/σ_0 (where σ_0 is the flow stress) as:

$$\frac{S_o}{S_{max}} = (0.825 - 0.34\alpha + 0.05\alpha^2)\left(\cos\frac{\pi S_{max}}{2\sigma_0}\right)^{\frac{1}{\alpha}} - (0.415 - 0.071\alpha)\frac{S_{max}}{\sigma_0} \quad (14)$$

The constraint factor, α represents the contribution of the material in the vicinity of the crack on the crack opening stress. The values for α of 1.1 for plane stress and 1.73 for plane strain were used by Newman et al [12] for an aerospace aluminum alloy. In that work, the crack opening stress ratio, S_o/S_{max} was calculated as a function of the crack size for various values of crack heights under completely reversed loading (S_{min}/S_{max} = **-1**) with an initial defect size. It was shown that the defect height greatly influences the closure behavior of small cracks. For defect heights above a certain critical value, the initial defect surfaces are found not to close even under compressive loading. The calculated crack opening stresses are not found to be influenced by the crack height. Newman *et al*'s results [12] show that S_o/S_{max} goes from a minimum value of –1 at the minimum crack size and then crosses the zero value and extends into the positive region with increasing crack sizes and asymptotically approaches the value corresponding to the large cracks.

V. RESULTS AND ANALYSIS

For this assessment, the material A533B was selected for which the long crack threshold (ΔK_{th}^{l}) is 8 MPa $\sqrt{m}$. The material has an assumed endurance stress range ($\Delta\sigma_e$) of 1000 MPa (endurance limit of 500 MPa), the value of the distance parameter "*d*" from equation (3) is calculated to be 70 µm or 0.07 mm. With these values the limiting stress range for fatigue failure *(ΔS)* as a function of crack size (a) is obtained using equation (4). The corresponding values using the approach due to El Haddad et al [4] are also obtained using equation (5). A comparison of the two methods is shown in Figure 1.

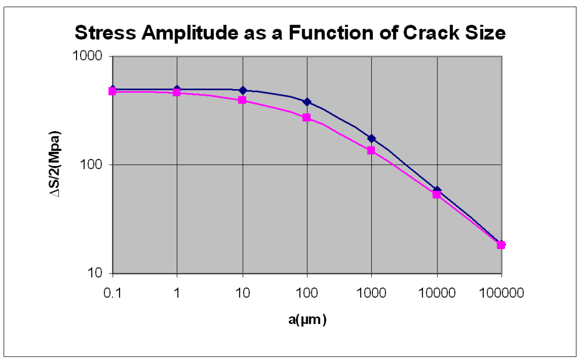

Figure 1. A Comparison of two methods - Upper Curve (El Haddad, [4]), Lower Curve (Roche, [13])

Table I. Stress range vs. number of cycles to failure (no crack closure)

Stress Range, ΔS, Mpa	Initial Crack Size, m	Cycles to Initiation	Cycles to Propagation	Total cycles
1000	0.0000204	773000	2095	775095
1100	0.0000168362	211409	1906	213315
1200	0.0000141471	63839	1721	65560
1300	0.0000120543	21303	1551	22854
1400	0.0000103938	7732	1399	9131
1500	0.00000905415	3015	1264	4279
1600	0.00000795775	1252	1146	2398
1700	0.00000704908	549	1042	1591
1800	0.0000062876	252	951	1203
1900	0.00000564317	121	870	991
2000	0.000005093	60	799	859

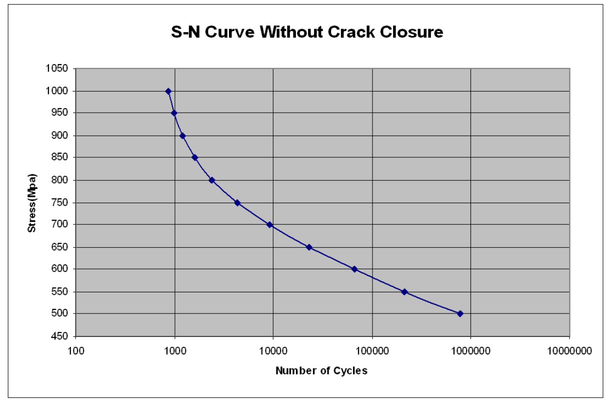

Figure 2. New S-N curve for SA 533 Gr. B without consideration of crack closure

For the material A533B, the proportional results for aluminum alloy were assumed to be applicable. For Smax /σ0 of 0.6, and a completely reversed stress (S_{min} / S_{max} = -1), the crack opening stress was calculated using equation (14) for values of α between 1.0 and 2.0. It was found S_o/S_{max} stays practically constant for large cracks at this stress level with a value of 0.11. Using this value in equation (13) the following relationship is obtained:

$$\Delta K_{eff} = 0.445 \Delta K \quad (15)$$

Using the square relationship to estimate Δ_{Jeff}., the following expression results

$$\Delta J_{eff} = (0.445)^2 \Delta J \quad (16)$$

The crack growth rate, da/dN is now expressed using equations (10) and (16) as:

$$\frac{da}{dN} = 2x10^{-4} (\Delta J)^{1.587} \quad (17)$$

The numbers of cycles (N_i) required to initiate the crack and grow it from zero to a_{th} stays the same as calculated in equation (9) for the no closure case; however to calculate the number of cycles (N_p) to propagate a long crack from a_{th} to *d*, equation (17) is used in place of equation (10). The resulting equation for *N* is similar (but not identical) to equation (12) and is not reproduced here.

The associated calculations are shown in Table II. The corresponding S-N curve is shown in Figure 3.

Table II Stress range vs. number of cycles to failure (crack closure)

Stress Range, ΔS,	Initial Crack Size, m	Cycles to Initiation	Cycles to Propagation	Total cycles
1000	0.0000204	773000	31419	804419
1100	0.0000168362	211409	28595	240004
1200	0.0000141471	63839	25812	89651
1300	0.0000120543	21303	23261	44564
1400	0.0000103938	7732	20980	28712
1500	0.00000905415	3015	18973	21988
1600	0.00000795775	1252	17189	18441
1700	0.00000704908	549	15633	16182
1800	0.0000062876	252	14259	14511
1900	0.00000564317	121	13060	13181
2000	0.000005093	60	11985	12045

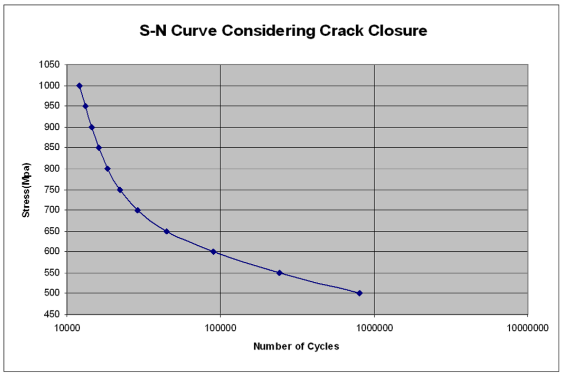

Figure 3. New S-N curve for SA 533 Gr. B considering crack closure

VI. FURTHER RESEACH

Further research in this area will incorporate several new concepts in the area of the theory of critical distances as outlined by Taylor [16].

VII. CONCLUSION

Most of the failures in the industrial world can be attributed to fatigue and is based on the response of the structural materials to alternating loads that are applied during service. The S-N curve, a graphical representation of the cyclic loading, is a plot of stress amplitude versus the number of cycles the material goes through before it fails. The greater the number of cycles in the loading history, the smaller the stress the material can withstand without failure. In keeping with the modern design philosophy of defect tolerant design, this work recognizes the presence of cracks in structural components. A rational process combining the nucleation of a crack on an assumed smooth surface (from a zero to the threshold value of crack length) and its subsequent growth to a characteristic dimension is assumed to govern the phenomenon of fatigue crack initiation. The number of cycles to initiate a crack is a feature that has been directly linked to the microstructure, and forms an important contribution of this study. By making use of an assumed crack initiation law and long crack growth law, the S-N curve for a structural material has been obtained. The curves have been obtained for the case of no crack closure as well as for the case of crack closure. The crack closure effects have been determined to be quite significant. The combined effect of the initiation and growth of small cracks along with crack closure constitute a correct representation of the fatigue damage. This work therefore provides reasonably accurate estimates of fatigue lives, and should be an invaluable tool to the designers of industrial machinery and equipment. Although most of the discussion in this work is directed to a typical pressure vessel steel material, the S-N curves for other materials can be obtained using a very similar approach.

NOMENCLATURE:

a	crack length, m
a_{th}	threshold crack size, m
d	distance parameter due to Roche (1990), m
E	modulus of elasticity, GPa
J	J-integral, N/m
ΔJ	J-integral range, N/m
K	stress intensity factor, MPa √m
K_{max}	maximum stress intensity factor, MPa √m
K_{min}	minimum stress intensity factor, MPa √m
K_o	crack opening stress intensity factor , MPa √m
ΔK_{th}	threshold stress intensity factor range, MPa √m
ΔK_{thl}	long crack threshold stress intensity factor range, MPa √m
ΔK_{eff}	effective stress intensity factor range, MPa √m
l_0	distance parameter due to El Haddad et al (1980), m
N	number of cycles
r	radial distance from the crack tip
S_{max}	maximum stress, MPa
S_{min}	minimum stress, MPa
S_o	crack opening stress, MPa
ΔS	stress range, MPa (remote from crack)
α	constraint factor
σo	flow stress, MPa
$\Delta\sigma$	stress range, MPa
$\Delta\sigma e$	endurance limit stress range, MPa

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