Theoretical and empirical comparison of coupling coefficient and refractive index estimation for coupled waveguide fiber

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ABSTRACT

Power transmission and coupling of coupled waveguide fibers are affected on the coupling coefficient. The coupling coefficients obtained in experimental results vary in widely. The coupling coefficients have a function of separation of fiber axis and refractive index of core and cladding. For the empirical formula, coupling coefficient is calculated from experimental result of coupling ratio distribution from 1% until 75%. Theoretically, the coupling coefficient has a dependence of some parameters resulting in a sinusoidal curve. Both empirical and theoretical formulae are compared to obtain new phenomena of the refractive index of fibers after fusion.

INDEX TERMS

Coupling coefficient, Coupling ratio, Refractive index, Single mode fiber

I. INTRODUCTION

Development and fabrication of single mode fiber couplers widely expands either for a tunable filter or an optical waveguide switch. These multi-purpose coupling fibers can be used in telecommunications or as device sensors [1], [2]. The fiber couplers fabricated control power transmission from one fiber to another by splitting it as a junction. However, the coupling fiber fabrication is complicated since the electric field and power are affected by the structure and geometry of the fiber itself. Therefore, the development of the fiber coupler for industrial application still continues to improve the transmission and reception of the output information.

One of the main phenomena occurring to couplers is the coupling of mode in space [3], [4], which contributes to power propagation along coupled fiber and is the coupling coefficient. The coupling coefficient can be expressed as an effective power range transmitted to another fiber. The separation of two fibers is significant in coupling coefficient because it determines the effective power transmission to another fiber. Even though the determination of coupling coefficient for a practical directional coupler is difficult, by evaluating the channel waveguide modes and observing the fiber geometry, some calculation of coupling coefficient range can be obtained. The electric wave propagates along a cylindrical fiber with a dielectric "weak" perturbation and the variation of mode amplitude is "slow", meaning it satisfies the condition of $|d^2/dz^2 A_k| << |\beta_k\ d/dz\ A_k|$ with a parabolic approximation where β_k, z, and A_k are respectively propagation constant, direction propagation and amplitude of the wave [5].

Power transmission and coupling depend on a distribution of the coupling ratio having a fractional power, which mainly occurs at the coupling region. At the center of the coupling region is the main coupling region where the power propagation can split from one core to another at the nearest distance of the coupled fibers. Power splitting depends on the coupling ratio and coupling coefficient. The value of the coupling coefficient can be determined by calculation and theory. However, the coupling coefficient has some parameters involved such as the refractive index of core and cladding, a separation of fiber at fusion, and a coupling length, which is parametric function dependence. The coupling coefficient has a wide range of values, not only in experimental result, but also by the theoretical calculation. This paper describes the coupling coefficient and refractive index, which are obtained experimentally from the coupling ratio distribution. A comparison with the theoretical calculation to determine a relationship between refractive index and separation fiber axis at the center of the coupling region after fusion is also carried out.

II. COUPLED FIBER AND COUPLING COEFFICIENT

The waveguide-carrying electric field is a single mode fiber (SMF-28e®) and is coupled by two fibers with a similar geometry. The electric field structures are homogeneous, iso- tropic materials with a very small gradient of refractive index along its propagation. SMF-28e® fibers are heated to a temperature range of 800-1300°C. Consider a coupled identical single mode fiber 1X2 splitting one source to become two transmissions as Y junction as illustrated in Figure 1. The total electric field together with mode propagation ε_a and ε_b at z axial length is a sum of electric field at the output junction expressed by [5]:

$$E= A(z)\, \varepsilon_a\, e^{i[\omega t - (\beta a + kaa)z]} + B(z)\, \varepsilon_b\, e^{i[\omega t - (\beta a + k\, bb)z]} \quad (1)$$

The change in amplitude is derived as follows:

$$\left.\begin{aligned} dA/dz &= -i\, \kappa_{ab}\, \mathrm{B} e^{i2\,\delta z} \\ dB/dz &= -i\, \kappa_{ba}\, B e^{-i2\,\delta z} \end{aligned}\right\} \quad (2)$$

where $2\delta = (\beta_a + \kappa_{aa}) - (\beta_b + \kappa_{bb})$

For the single input at guide a is $\{A(0) = Ao\ ,\ B(0)=0\}$, and a solution is given by:

$$\left.\begin{aligned} A_m(z) &= A_o e^{i\,\delta z}\{\cos[(\kappa^2+\delta^2)^{1/2}z] - i\delta\,(\kappa^2+\delta^2)^{1/2}\sin[(\kappa^2+\delta^2)^{1/2}z]\} \\ B_m(z) &= -\,i\,A_o e^{-i\,\delta z}[\kappa/(\kappa^2+\delta^2)^{1/2}]\sin[(\kappa^2+\delta^2)^{1/2}z] \end{aligned}\right\} \quad (3)$$

In terms of powers, $P_a = A^* A$ and $P_b = B^* B$ for two waveguides. In the case of $\kappa_{ab}=\kappa_{ba}=\kappa$, by assuming κ_{aa} and κ_{bb} are very small values and waveguides are not too small then:

$$\mathrm{P_a(z) = P_o - P_b(z)}$$
$$P_b(z) = P_o\, \kappa^2 /(\kappa^2+\delta^2)\, \sin^2[(\kappa^2+\delta^2)^{1/2}\, z] \quad (4)$$

$P_o = |A(0)|^2$ is an input power to guide and complete power transfer at a distance of $z=m\pi/\kappa$; $m=0,1,2,\ldots$, for $m=1$, $z=L_c= \pi/\kappa$. L_c is the coupling length in millimeter unit. The axial length is periodically changed by a coupling ratio [6]. $P_b/(P_a+P_b)$ and $P_a/(P_a+P_b)$ are defined as coupling power and transmission power respectively. The wave propagates as a sine and cosine wave where $\kappa = \surd(\delta^2 + \kappa^2)$ is the coupling coefficient and δ is the phase mismatch factor defined as $(\beta_1 - \beta_2)/2$. If a laser diode source wave length λ =1310nm is used which is related to the normalized frequency as $V= 2\pi a/[\lambda\, (n_1^2 - n_2^2)^{1/2}]$, then coupling coefficient, κ can be defined as follows [7],

$$\kappa = \frac{(2\Delta)^{1/2}}{a}\, \frac{u^2}{V^3}\, \frac{K_o(wd\ /a)}{K_1^2(w)} \quad (5)$$

where K_o and K_1 are the zeroth and first order of Hankel function. Normalized lateral phase constant and normalized lateral attenuation constant are u and w respectively. The symbol of Δ is defined by $(n_1^2 - n_2^2)/2n_1^2$, a is the core radius and d is the separation fiber axis between cores. A simple empirical relationship is used to calculate the value κ [8] which is as follows:

$$\kappa = (\pi/2)\, (\surd\delta/a)\, \exp\,[-\,(A + B\, đ + c\, đ^2)] \quad (6)$$

where
$$\begin{aligned} A &= 5.2789 - 3.663V + 0.3841V^2 \\ B &= -7769 + 1.2252V - 0.012V^2 \\ C &= -0.0175 - 0.0064V - 0.0009V^2 \\ \delta &= (n_1^2 - n_2^2)/n_1^2 \qquad ;\ đ = d/a \end{aligned}$$

n_1 and n_2 are core and cladding refractive indices respectively.

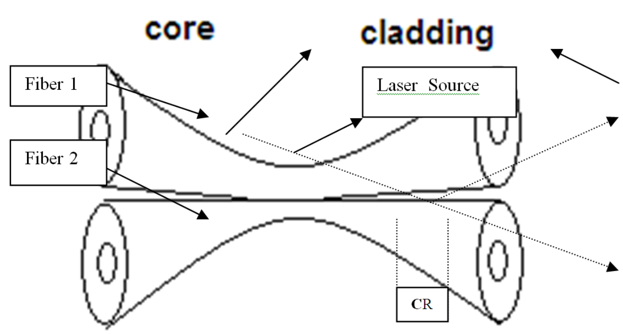

Figure 1. SMF-28e® coupling region

III. RESULTS AND DISCUSSION

In the fusion process, heated fibers are not homogenous in changing their structures and geometries at the coupling region. These changes are complicated because refractive indices and fiber geometries fluctuate toward the coupling ratio. They tend to decrease exponentially along fibers from one edge to the center of the coupling region and again increase to the other end. The wave and power propagation change partially along the coupling region although the conservation of power is independent of position. For simplicity, it is assumed that the results concentrate at the center of the coupling region with a homogeneous structure and geometries. Fusion fiber diameters at the coupling region decrease to 75-85% where the previous core and cladding diameters were 8.2μm and 125μm respectively. Measurements of fused coupling fibers geometry are observed by a microscope. The coupling length can be computed provided that $\delta=0$ where it has equal phase velocities in both modes and $\kappa/(\kappa^2+\delta^2)$ is a fraction of power exchanged as stated by equation (4). In this calculation, κ is fixed at 0.903 where the core and cladding refractive index is n_1= 1.4677, n_2=1.4626. At the same time, coupling length increases toward the coupling ratio. This occurs at the coupling region by a few ms to reach complete coupling power where time is proportional to the coupling ratio.

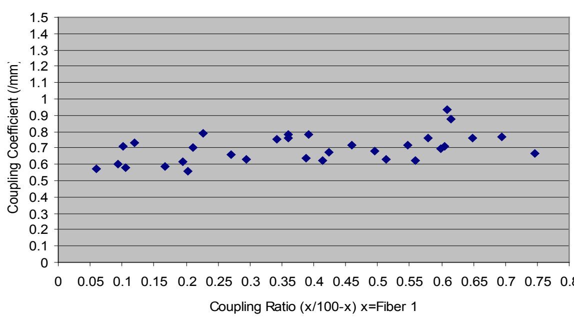

Figure 2. SMF-28e coupling coefficient at the center of coupling region after fusion where coupling length and refractive index vary

There is, however, a discrepancy of coupling length through measurement and fixed κ over a range of 0.25mm. The separation between core diameter d cannot be measured precisely, but for calculation purposes is set at 10μm [8]. Power losses during fusion also contribute to the coupling ratio range while for the calculation of fixed κ, it not considered. In terms of the coupling ratio, the experimental results describe how the power transmitted at the coupling region had a larger coupling length than the calculated value. This coupling coefficient is shown in Figure 2. A coupling coefficient in the range of 0.6-0.9/mm is assumed for the power transmission and coupling occurs during fusion. Some parameters fluctuate during fibers twisting and fibers heating. The refractive index change cannot be controlled during measurement. The refractive index dependence toward the coupling coefficient is shown in Figure 3 before and after fusion curves. Before fusion, refractive index is higher than after fusion [9].

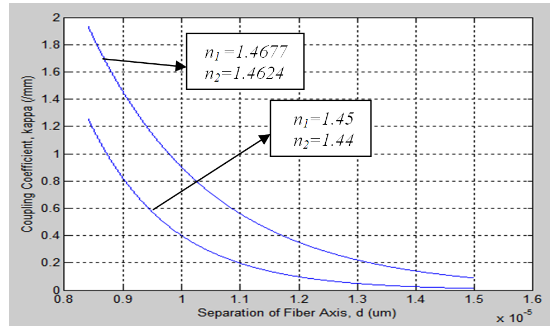

Figure 3. Empirical formula to compute coupling coefficient of SMF-28e coupler heated by torch flame at the center of coupling region.

The two curves represent a boundary of fiber refractive index occurring when the fibers are fused at n_1=1.45 and n_2=1.44. It is expected that κ is within the range between two curves. For the range of 0.6-0.9/mm, the fiber separation of 10-10.86μm, n_1=1.4677 and n_2=1.4624 are as shown in Figure 3. Comparing the cladding diameter obtained experimentally in the range of coupling ratio of 1-75%, d is between 9-17.5μm. To verify that the last refractive index is a boundary range of two curves, it is given a mean separation of fibers d=10-10.86μm. Figure 4 shows the δ decreases rapidly from 0.007-0.01 with a high gradient until the coupling coefficient reaches approximately 0.3. As expected, the refractive index does not reach n_1=1.45 and n_2=1.44 when fibers are fused since the power transmission that keeps flowing along the fibers are higher than κ must be reasonable. By setting κ = 0.9-0.6/mm, the refractive index change $(n_1^2-n_2^2)/n_1^2$ is in the range of 0.0086-0.092. Implicitly, this number is δ, which the refractive index of both core and cladding is after fusion. Evaluation of δ, n_2/n_1 is in between 0.9956-0.9953 assuming that n_2/n_1 proportionally decreases over κ then typically n_2=1.4577-1.4556 and n_1=1.4640-1.4623.

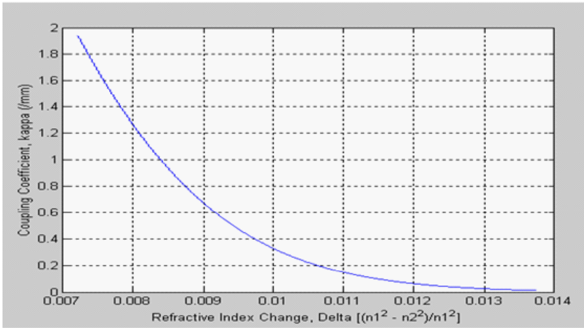

Figure 4. Refractive index decreases due to fused fibers for, n core=1.4677-1.45, n cladding=1.4624-1.44

The two gradients of 0.9956 and 0.9953 shown in Figure 5 represent the relationship of refractive index with coupling coefficient at 0.9-0.6/mm. The two curves distance is narrow compared to small changes of refractive index by factor 10^{-3}, while κ extends widely as expected from Equation (5) and (6). The upper line is a gradient of n_1/n_2 before fusion and after fusion n_1 and n_2, is attached in the box. The change of refractive index from previous to the latter value together with $\Delta\kappa$, is 0.24-0.36% for core and 0.31-0.45% for cladding. This shows that the coupling coefficient depends upon the refractive index, but it is not *vice versa*.

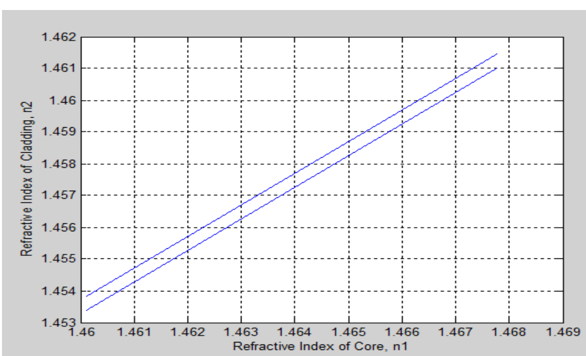

Figure 5. Gradient of refractive index for kappa 0.9/mm (upper line) and 0.6/mm

Comparing Figure 3 and 4, the coupling coefficient is mainly affected by fiber separation by a factor 10^{-1} compared to the refractive index, which is by a factor 10^{-2}. Decreasing the refractive index reduces wave propagation along fibers in time and position with that mode widely travelling to both claddings, whereas decreasing core fiber separation effects allows the coupling ratio to be achieved earlier.

According to Equation (6), the value of κ depends on d by an exponential factor and δ, rather than a refractive index of n_1 and n_2. V and a are assumed to be constant, but actually they do change during fusion together with a wave number and propagation constant due to changes in fiber geometry. This description can be observed in Figure 6. Since d is proportional to κ, then increasing d at fixed δ increases κ, although separation fibers are not suitable. For a given core spacing, the coupling strength increases with the wavelength. This is due to the fact that the field extends deeper into the cladding structure and typical values are in the range of 0.1 to 2 mm^{-1} for core spacing in the range 4 to 15µm [7]. It also shows a strong decrease when the fiber core spacing increases, as expected from Equation (5) by assuming that the interaction length is negligible as compared to the radius of the curvature of the fibers and the wavelength. On the other hand, δ decays smoothly at lower d rather than at higher d. The refractive index difference between core and cladding is very important in order to support power transmission as a coupling coefficient is increased. However, if it does occur, the power and wave radiate too much into the cladding. On the contrary, it is not the case for a commercially-available single mode fiber. It is necessary to obtain a higher coupling coefficient to have optimum d and δ. Both parameters cannot be controlled easily during fusion until the coupling ratio preset value is achieved.

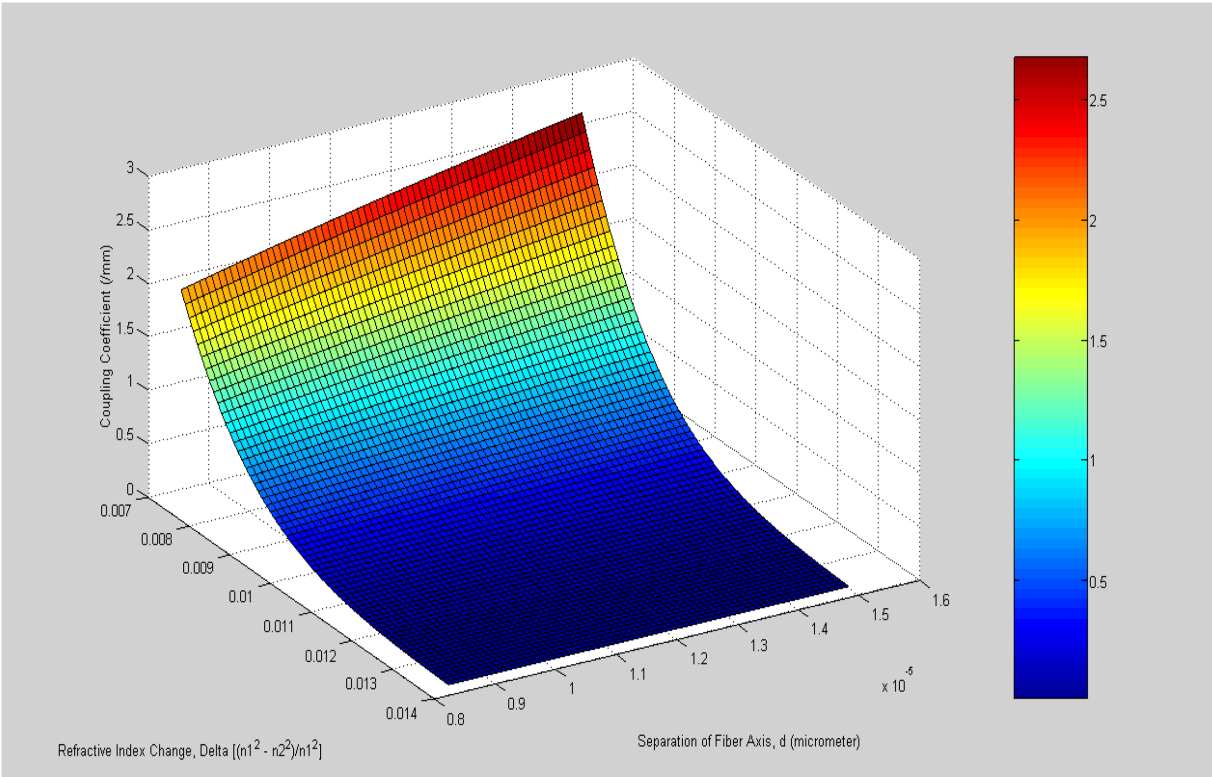

Figure 6. Coupling coefficient of SMF-28e for wide range of δ and d. n_1=1.4677-1.45 and n_2=1.4624-1.44

Similarly, in theoretical calculation according to Equation (5), the coupling coefficient can be computed as shown in Figure 7 and 8. The κ value satisfying δ and d is as follows: κ value is 0.5485-0.7151/mm where d=1.278-1.2845x10^{-5}m and $\Delta\delta$ = 0.0061. For κ value is 0.9417-1.0061/mm where d=1.2934-1.296x10^{-5}m and $\Delta\delta$ is 0.0061. The κ value is expected between 0.6-0.9/mm, but it is found at 0.5485-1.0061/mm then n_1=1.4541-1.4543 and n_2 = 1.4452-1.4454. The gradient between cladding and core n_2/n_1 is 0.99387 and 0.99388.

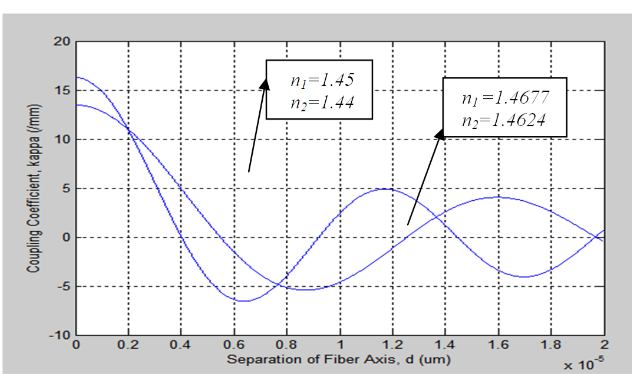

Figure 7. Coupling coefficient of SMF-28e coupler heated by torch flame

Wherever the two curves intersect, we have the same refractive index at certain separation fiber for both core and cladding. This occurs since the change of d, n_1 and n_2 attain the same κ. It also expresses that refractive indices are proportional to κ. The curve calculation is in a wide range of d starting from zero then κ is much higher at the core size, while d starts 10-10.86μm, and then κ sinusoidal changes and decays earlier to less refractive index toward d. For each point of d, it has found a different value of n.

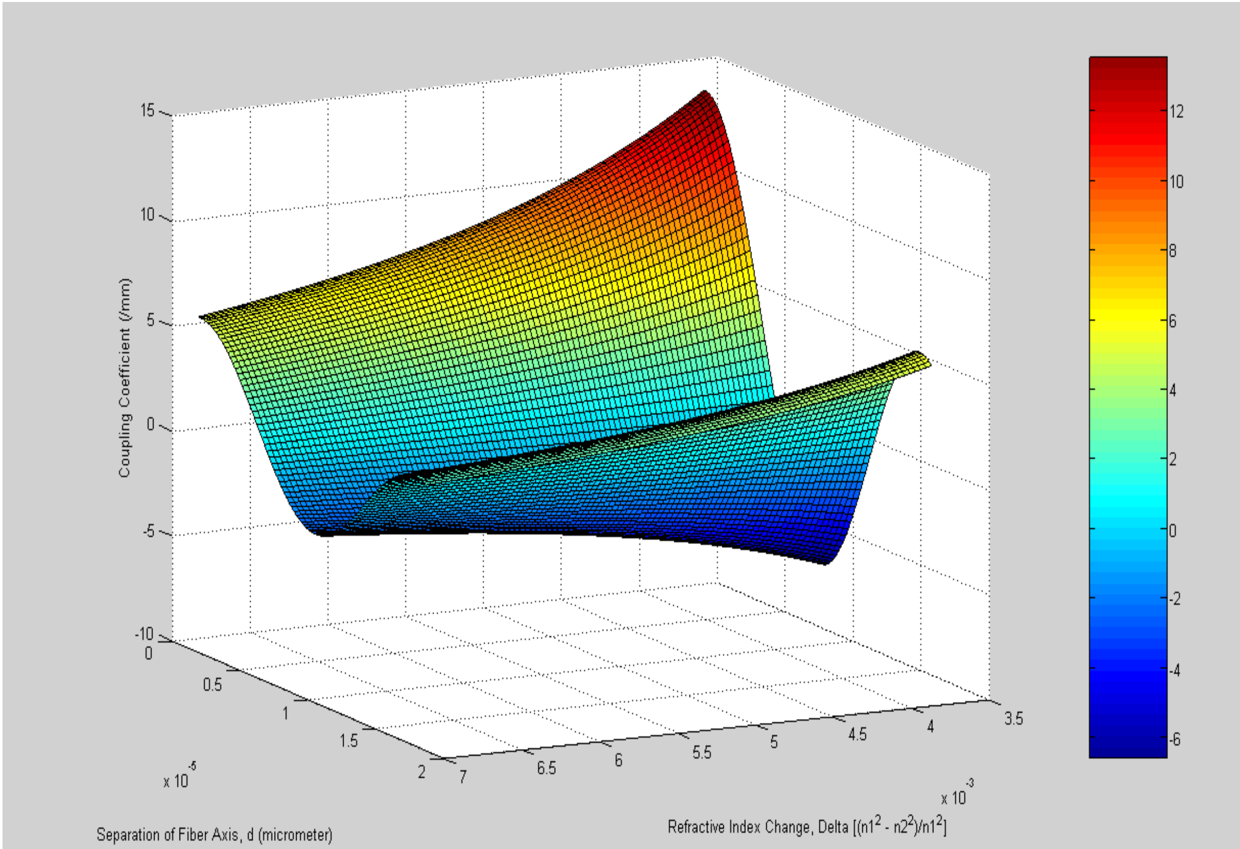

Figure 8. Refractive index decreases due to fused fibers for, n core=1.4677-1.45, n cladding=1.4624-1.44

IV. CONCLUSION

A coupling coefficient based on distribution of the coupling ratio has been obtained. It is in the range of 0.9-0.6/mm corresponding to the determination of the refractive index by the empirical equation to core and cladding, respectively n_1=1.4640-1.4623 and n_2=1.4577-1.4556, where n_2/n_1 is 0.9956 and 0.9953. Calculated κ values are 0.5485-1.0061/mm corresponding to n_1=1.4541-1.4543 and n_2=1.4452-1.4454 where n_2/n_1 is 0.99387 and 0.99388. The empirical equation shows the refractive index is less affected than the separation fibers to reach coupling coefficient of 1-75% rather than theoretical calculation for the same coupling ratio. On the other hand, the theoretical calculation expresses the coupling coefficient in a wide range and is proportional to a lower value of refractive indices. This means that n_2/n_1 for empirical calculation is higher than a theoretical calculation and this difference occurs when reaching the coupling ratio. No magnitude losses are assumed in theoretical calculation, while in the empirical calculation it is only imposed by power.

V. ACKNOWLEDGMENT

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