

CONCEPT 1: A Guide to Primary Mathematics
A Visual Design Problem

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Title of Thesis CONCEPT / A GUIDE TO PRIMARY APPROPRIATIONS
A VISUAL DESIGN PROBLEM

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1.0 Background

During the first portion of my graduate study at the Rochester Institute of Technology, I chose to concentrate on the visualization process in communication design, the creation of visual images for solving two-dimensional graphic design problems. During this study, I came to realize that a particular group of visual aesthetic values, namely formal design values, offer the designer the foundation for all his imagery and help him control his form-making, thus improving its effectiveness. The study included discovering the critical importance of line and its interval relationships, mass and quantity relationships, color and its visual perception, the dynamics of image translation, and the effectiveness of organizational graphic systems. So exciting and profound was this study that I decided to continue my exploration in the thesis format. This thesis then, is a result of an intensive two-year study of formal design values in their purest sense.

Next to consider was the thesis topic itself and here

too I chose to diverge from the norm a bit. Rather than apply the formal design values to a graphic design problem in an adult environment, I selected the world of the child as the testing area. Can these aesthetic values function in a child's environment or better yet, through their introduction, can they offer the child a new perspective on visual aesthetics? Can they begin to establish a visual foundation for the child and in doing so, initiate a new level of aesthetic values for future communities?

It is the visual answer to these questions that I present in the following thesis.

2.0 Statement of Intent

The intent of this thesis is to explore the application of formal design values in a realistic problem-solving situation.

The problem deals with the education of pre-school or first level elementary school children in the area of primary mathematics.

Part one of the Pre-program prepares the child for understanding the general perceptual principles of mathematics.

Part two deals with more specific primary mathematics goals. These goals require the child's comprehension of:

Form: the ability to recognize and understand
 the numbers 1 - 100.

Sequencing: the ability to recognize and place in
 order the numbers 1 - 100.

Multiples: the capability of counting by groups of
 2, 5, and 10.

- Grouping:** the understanding of the simple grouping of objects and their addition to or subtraction from the group (primary numbers, 1 - 9, only).
- Recall:** the basic recall of the addition and subtraction skill.

It should be noted here that CONCEPT 1, as this program would come to be called, does not pretend to offer the ideal solution for a primary mathematics program. Rather, its purpose is to suggest one reasonable concept direction, translated into a visually stimulating experience, with a foundation of formal design values. And because of this formal foundation, CONCEPT 1 has a secondary function. It begins to establish a basis for a child's visual literacy, a sensitivity to form and visual aesthetics which is many times misplaced in primary education.

Through the CONCEPT 1 model, the potential of applying the formal design values to other topics in the educational environment, will become apparent.

3.0 Research

Prior to the initiation of the visualization process, it seemed important to examine current primary mathematics programs. It was hoped that these studies could offer some insight into the educational process as well as act as a foundation to which the formal design values could be applied. Five separate texts from school systems in different sections of the United States were collected and examined. While each program varied in its format and imagery, final analysis was so similar that the conclusions can easily be made in unison:

1. The programs lacked any logical continuity from section to section; from understanding form to sequencing, from multiples to grouping.
2. The visualization of simple objects was arbitrary; that is, the form of the images did not clearly represent the objects. Rather, the images appeared to be an artist's interpretation of what he thought a child's interpretation of an object would be.
3. Color was applied arbitrarily rather than for a particular purpose.

4. The organization of information was complicated within the sections and within each page layout.
5. Mathematics concepts were not clearly conceived.

A series of meetings with a panel of first grade educators brought about similar conclusions.

Observing the existing texts, then, did little but reaffirm the need for a strong, visually coherent program of primary mathematics instruction.

How Children Perceive

The key link to the eventual development of CONCEPT 1 came with the realization that understanding how children perceive is critically important. Through Dr. Paul Kazmierski, Director of The Learning Development Center at the Rochester Institute of Technology, this became eminently clear. He set up a Basic Model of Thinking and Learning and explained how it affected a child's ability to learn.

A child receives a stimulus, perceives it, recalls it, either through past or present experiences, and re-

sponds to it on a perceptual or literal level. In the case of mathematics, a child will respond to the stimulus of $1 + 2 = ?$ with a "3" response, providing he has had the past experience of $1 + 2 = 3$. But within this perceptual level, that is his limit. He can neither interpret nor manipulate information, as adults are capable of doing.

At the next stage, the interpretative level, the child enters a plateau which is critical for the learning process to take place. Now he can manipulate and interpret information. For example, in the case of $1 + 2 = 3$, the child will respond with the answer "3" whether he is stimulated by the question of $1 + 2 = ?$ or the manipulated version of $2 + 1 = ?$ In other words, it is not until the child has reached the cognitive level of learning, that he is able to truly understand mathematical concepts. Previous learning is simply "rote" learning.

The time table for passage through the Model of Thinking and Learning varies from child to child according to his experiences. However, the average age for passage from the perceptual level to the

critical cognitive level is between 5 and 7 years, roughly the age of the first grader. It is conceivable then, that a first grader may not be ready to accept the basic concepts of mathematics regardless of the educational program, its form or content.

Indeed prominent psychologists in the field of child learning have found certain basic mathematics concepts incomprehensible to many children in this elementary age group. One psychologist, Jean Piaget, spent over 30 years, testing and studying children. His findings confirm: "Learning" for children who have not reached the cognitive level is "Memorizing and parrot-like learning."¹ In fact, many psychologists feel exposing a child to mathematics prior to his readiness to understand it may cause the child to resent it, a resentment that could remain with him indefinitely.

To confirm the conclusions that in today's classroom some children may not be prepared for formalized mathematics instruction, a first grade class at Leary

1 Evelyn Sharp, Thinking is Child's Play (New York: Avon, 1969), p. 27.

Elementary School in Rush, New York, was selected as a test group. A variety of Bender-Gestalt and Piaget tests were given to determine whether these children had, in fact, reached a cognitive level of comprehension. As hypothesized, the class response was mixed; some children were simply not prepared to understand basic mathematics concepts.

While these tests were not professionally administered and controlled, their results strongly supported the conclusions of Piaget and his colleagues. But most importantly, the tests pointed to a need for a two-part mathematics program: a pre-program of readiness for children who were not yet prepared for formalized mathematics instruction, and a more formalized program satisfying the traditional teaching objectives of basic mathematics.

Continued research uncovered an existing program which was designed to ready pre-school or pre-cognitive children for understanding mathematics. In her book, "Thinking is Child's Play, Evelyn Sharp outlines the program in detail. Comprised of a series of games, each game is based on the findings of Jean Piaget.

Through the experiences of the games, children more quickly learn important cognitive principles relating to primary mathematics, thus speeding up their inevitable jump from pre-cognitive to cognitive learning. This program so perfectly satisfies the requirements for the Pre-program to CONCEPT 1, that it was adopted as part of this thesis. For examples of the games, see "Pre-program to CONCEPT 1."

What Children Perceive

The critical stage of cognitive learning also affects the types of images to which a child will respond and recognize. An awareness of what children perceive in light of response and recognition became particularly significant when initiating the visualization process for CONCEPT 1.

Through research in a variety of texts, including The Visual World of a Child, and The Teaching of a Young Child, it was learned that the child who has matured into the cognitive phase of learning will, within limitations, recognize and respond to simple visual images in much the same manner as adults.

A good example of the visual sophistication of the cognitive child comes in an excerpt from, The Visual World of the Child:

"From the ages of 6 to 8 years, the perceptual structures have lost much of their rigidity; the child of this age can break these structures down into parts, abstract certain of their characteristics and attend to one part while disregarding the others. Perceptual units can be combined, each one serving as a whole to the smaller units of which it is composed and at the same time as a component part of the comprehensive structure."²

Within reason, then, a first grade child, who has reached the cognitive level of learning should have little difficulty in interpreting simple translated images or photographic images selected from a familiar internal or external environment. Many of these images comprise the program of CONCEPT 1.

2 Elliane Vurpillet, The Visual World of a Child (Los Angeles: International Universities, 1974), p. 123.

4.0 Pre-program to CONCEPT 1

Essentially, the pre-program consists of a series of games designed to assist the child in understanding three basic cognitive principles as defined by Piaget:

1. Classification: the ability to recognize likenesses and differences among objects and to group them.
2. Seriation: Arranging objects in a series according to some specified order.
3. Conservation: The understanding of mass or part/whole relationships.

The following four games have been selected from Thinking is Child's Play, to illustrate the idea of the Pre-program to CONCEPT 1.

Each game requires a series of colored circles, squares, and triangles, two of red, green, blue, and yellow in each of the shapes. In some cases, simple household objects are used.

Game 1

Purpose: To provide experience in classification by color.

Materials: The eight squares from the set of colored cards.

Spread the squares out in no particular order but with all of them plainly visible. Single out a red one.

"What color is this?"

"Put the two red ones together."

Repeat with each of the other colors. Then mix them all up again.

"Can you put the ones together that are alike?"

Comments: Children use color, shape, and size in classifying things around them. Color is the easiest of these three attributes, and the best place to start. Colors with decided and striking differences come first-- subtle gradations of shade and hue should not be used until later.³

3 Evelyn Sharp, Thinking is Child's Play (New York: Avon, 1969), p. 52.

Game 2

Purpose: To provide experience in classification by color.

Materials: The whole set of twenty-four colored cards. A small piece each of the same red, blue, and yellow plastic tape used in making the colored cards.

Spread out all the cards on one side of a fairly large working surface, such as a table or the floor. Stick a small cross of red tape on the other side. "Can you find all the red cards and put them over here?"

When the child has successfully done that, repeat with blue and then with yellow (identifying each sorting place with a cross of the appropriate color), leaving each group in place.

"What color are all these cards that are left over?"

Point out that he has now sorted the whole set by color.

Comments: In the first classification game all the cards were the same shape and the same size, differing only in color. Here they differ in more than one way but are classified according to the single attribute of color.⁴

4 Ibid., p. 55.

Game 3

Purpose: To provide experience in classification by shape.

Materials: The red squares, circles, and triangles from the set of colored cards.

Put the six red cards on the table, spreading them well apart. Single out a red circle.

"Can you find another round one like this?"

"These are circles. Put the two circles together."

Pick out a red square.

"Can you point to another one that's the same shape as this?"

As you say the word "shape", trace around the outside of the square with your finger.

"These are squares. Put the two squares together."

"These two that are left are triangles. They go together because they are the same shape."

Again trace around the outside of the card as you say "shape." Mix the cards up.

"Can you put the ones together that are alike?"

Comments: It is important to begin with cards all the same color in order to highlight the difference in shape. Use the words "square," "circle," and "triangle," but

always point to the appropriate card at the same time so that the child is not dependent on the name.⁵

⁵ Ibid., p. 57.

Game 4

Purpose: To provide experience in classification by two characteristics (color and shape) at one time.

Materials: The set of twenty-four colored cards. Take four squares, four circles, and four triangles, making sure that you have each shape in four different colors. Let the child sort them by shape. Put the triangles and the circles in two rows, but with the colors mixed.

"Can you fix them so red is under red, yellow under yellow, and so on all the way across?"

Add the row of squares (also with the colors mixed).

"Fix the squares, too, so the colors match."

When the cards are all arranged, start to play.

"Hide your eyes while I take one of the cards away."

Keep the one you took concealed.

"Can you guess which card is missing?"

Repeat several times; then reverse roles and hide your eyes while the child removes a card. Now you guess which one is missing.

Comments: In playing the guessing game, be sure to begin each time with the full arrangement so that only one card is missing at a time.

The child should identify the missing card by its two characteristics--"the red circle," for example. If this game is easy for the child, make it more challenging by mixing the cards up before you take one away. Can he guess what card you took? Does it occur to him to arrange the cards as before, so that it is easy to tell what is missing?⁶

6 Ibid., p. 100.

5.0 Outline of Visual Components

To most efficiently and effectively satisfy the objectives of CONCEPT 1, three primary components and a group of secondary components were established.

The primary components include:

1. A series of cards, numbering 1 - 100, housed in a loose leaf booklet.
2. A series of booklets dealing with the conceptual meaning of numbers 1 - 9.
3. A series of cards housed in a loose leaf booklet illustrating the addition and subtraction of the primary numbers 1 - 9.

The secondary components are comprised of two parts:

1. A series of posters to document the conceptual meaning of numbers 1 - 9.⁷
2. A workbook designed to help the instructor determine the child's level of understanding within CONCEPT 1.⁷

⁷ Suggested but not included in this thesis.

6.0 Primary Components of CONCEPT 1

6.1 Understanding the Numbers 1 - 100

Numbers can be viewed as increasing intervals. That is to say, in the traditional process of studying and learning primary mathematics, numbers are viewed logically from 1 - 10 or 1 - 100 in an increasing fashion: two is greater than one, three is greater than two and so on. It seemed appropriate then to try to capture this process in the visual approach of CONCEPT 1.

The visual solution to one of the program's objectives, the understanding of numbers 1 - 100, their form and their sequencing, laid the foundation which was to become the basis for all imagery in CONCEPT 1: the increasing line interval. A series of cards, 5" X 7" in format, were developed and each was given a number designation from 1 - 10. Each card was divided into black and white vertical bands of an increasing nature. When the bands were physically counted, their total was equal to the card's number designation. Because of their unique increasing quality, the black bands moving from thin to thick and left to right, were

identical in number and size to the white bands moving from thin to thick in the opposite direction. For example, in the case of the number 5, the 5" X 7" card was divided so that there were five primary black bands, increasing in thickness from left to right, as well as a set of white bands, increasing in the opposite direction. Therefore regardless of whether the child counts the black or white bands, he obtains the same designated number. And the child has a physical and visual representation of the number as it is discussed.

Another reason for the use of increasing interval bands as opposed to the regular and equal bands is simply the visual activity and interest the increasing bands create.

Positioned on the reverse side of each card is a 4 1/2 inch high numeral, representing the designated number of the card. Its purpose is to begin to familiarize the child with the form of the number. Along with the numeric designation is a location chart which illustrates and emphasizes the position of the particular number in relationship to other numbers from 1 - 100. To reinforce the increasing interval concept, space between the columns in the chart increases from left to right,

top to bottom, and the numbers become bolder and larger in size as they approach 100.

Color coding was then introduced to solve the "multiples" phase of teaching primary mathematics: counting by 2's, 5's, and 10's. By grouping together similar colored bands, imprinted on the cards, a child can easily discover which numbers belong to the same set.

Finally, as mentioned earlier, this stage of CONCEPT 1 is not only concerned with the understanding of the numbers 1 - 10, but also with the realization of every number from 1 - 100. Interestingly enough, the visual direction used for the numbers 1 - 10 could be applied for numbers 1 - 100, since the primary numbers are the foundation for all others. For example, the number 5 and the number 55 are similar in that the number 55 holds the same relative position in its sequence from 50 - 60 as 5 does in its sequence from 0 - 10. Because this program and many other primary mathematics programs present and study numbers in their relative groups of 10, the simplicity of reintroducing the increasing interval of 5, for example, on all "5" related cards

became apparent. Hence, each of the numbers 5, 15, 25, 35, 45 and so on use the basic increasing interval of five for their visual designation.

The opposite side of the card also follows the format of cards 1 - 10: the large numeral depicts the numeric designation of the card, the location chart calls out the number's position in relation to the first one hundred numbers and a color code makes the grouping of sets 2, 5, and 10 easy.

Summary

This phase of CONCEPT 1 helps the child understand:

1. The form of numbers 1 - 100.
2. The sequence of numbers 1 - 100.
3. The grouping of sets 2, 5, and 10.

6.0 Primary components of CONCEPT 1

6.2 Understanding the Meaning of Primary Numbers

While the first segment of CONCEPT 1 satisfies the general requirements for understanding numbers 1 - 10 and 1 - 100, it does not offer the child an intensive study of the conceptual meaning of the primary numbers upon which the numeric system is based. This understanding is critical and for this reason a series of concept booklets were created to illustrate the unique concept of each primary number. The concept booklet for the number 3 is used to illustrate this segment of the program.

Contained within the booklet "Understanding the Concept of Three," are a series of images whose inherent quality represents the meaning of the number 3. For example, the triangle was selected because it is three-sided, an inherent quality of 3. As a secondary reinforcement, three triangles of increasing size are used. The images are organized within an increasing interval grid, based on the axes created by the vertical bands found on the cards of 1 - 100

and described in the previous section. A grid is an organizational system which allows the designer to incorporate within a common format a variety of objects and forms. Another spread in this booklet uses a formal translation of a traffic light to illustrate the concept of 3. Through the visual translation, an object becomes a new graphic entity, where all the forces of visual aesthetics can be combined. Like the triangle, the traffic light contains a clear, inherent quality of 3. Using three traffic lights further supports the concept as does the introduction of three colors: red, yellow and green.

Still another layout offers a photograph of a bridge comprised of triangular segments. To bring emphasis to the three-sided elements within the bridge, a translation was superimposed over the actual photograph. The result was an illustration of the simple triangle in a complex external environmental situation.

Summary

This phase of CONCEPT 1 helps the child to understand the conceptual meaning of numbers 1 - 9.

6.0 Primary Components of CONCEPT 1

6.3 Understanding the Addition and Subtraction of Primary Numbers

The increasing vertical bands of black and white, created as a visual basis for the first portion of CONCEPT 1, Understanding the Numbers 1 - 100, became the foundation for the addition and subtraction booklet. However one significant change was made. The numeric designation for the card which was once positioned on the back of the card, was now integrated within the vertical bands on the front side. Thin black and white lines in a 50 - 50 ratio were incorporated to define the numeral and these new integrated cards became the "playing" cards for the addition and subtraction "game."

Symbols for addition and subtraction were also integrated within the vertical bands and each symbol was assigned a color. This coding makes it easy to distinguish the operation of addition from subtraction.

A loose leaf booklet houses the cards, arranged in

an order so that every possible addition and subtraction combination for numbers 0 - 9 is illustrated. For this thesis the section of the booklet concerned with the addition and subtraction of the number 5 is offered as a typical example. A more detailed visualization of this booklet and its operation is available in section 8.3.

Summary

This phase of CONCEPT 1 helps the child understand the addition and subtraction of primary numbers.

7.0 Secondary Components of CONCEPT 1

To reinforce the activities within the primary segments of CONCEPT 1, two additional components are recommended. One is a series of posters which visually summarizes the content of the booklets teaching the conceptual meaning of the primary numbers. These posters would be displayed in the classroom upon the introduction of a new number concept. Each number, therefore, would have its unique poster around which discussions with the children could center.

The second component this program recommends is a student workbook, used in much the same fashion as current first grade student "texts" are used. Its role is to assist the instructor in determining whether the child is comprehending the various mathematics concepts of this program. It also acts as a physical testing and exploration source for the child as he begins his study of primary mathematics.

Summary

The secondary components of CONCEPT 1 help the child understand the conceptual meaning of numbers 1 - 9 and assist the child and instructor in determining the child's level of comprehension within the program.

8.0 The Identity Program

The name, CONCEPT 1, grew out of the essence of what this primary mathematics program really is. It is the first in a series of many math programs a student will experience. It is based on primary numbers beginning with 1. And finally it reflects the concept that numbers are increasing intervals of 1. The name CONCEPT 1 seemed appropriate.

As well, a visual identity was established for two reasons:

1. To offer distinction among the components of the program.
2. To offer an overall visual unity to the program.

The primary element for the identity came from a part of the program already completed. It was the card representing the number 1 which was used in the addition and subtraction segment, that became the graphic representation for CONCEPT 1. It was applied in black to the cover of the binder, housing the number cards of 1 - 100 and in red and blue to the

binder for understanding addition and subtraction. The series of booklets designed to discuss the conceptual meaning of number 1 - 9 did not utilize the "number 1" identity element. Rather, the cover of each booklet changed with its content. For example, on the cover of the booklet for "Understanding the Number 3," the number 3 card from the addition and subtraction booklet was used as the identity. However, because of the strong family of images created within the visual program, each of the booklets tied together and related to the overall identity concept.

9.0 Visualization

In the Visualization segment of this thesis, key components of CONCEPT 1 have been illustrated for better understanding of the program. Where necessary, a more detailed verbal explanation accompanies the visual; otherwise simple brief statements are used. It should be noted that each of the sections within this Visualization segment, correspond to preceding sections of this thesis for easy reference.

9.0 Visualization

9.1 Understanding the Numbers 1 - 100.

1

Initial line and numeral integration studies, where ground line weight and figure line weight vary in each segment.

2

Initial line interval studies for the front surface of cards 1 - 100. This study explores line mass as a way of conveying the increasing quality of numbers.

3, 4, 5

Front and back side of cards 1 - 9. The gray bands on cards 2, 4, 6, and 8 represent color bands of blue, indicating that these numbers are part of the set of 2. The gray band on the 'five' card is yellow, the color code for numbers in the set of 5.

6

Front and back side of cards 54, 55, and 56, given the same graphic treatment as numbers 1 - 9.

7

Color reference for the front and back side of the 2, 4, and 6 cards for the series of cards, 1 - 100. Note the increasing quality of the color bands as they move from the 'two' card to the 'six'. The blue color indicates that these cards are of the same set of 2.

8

Color reference for the front and back side of the 4, 5, and 6 cards from the series of cards 1 - 100.

9.0 Visualization

9.2 Understanding the Meaning of Primary Numbers

9

Initial studies for the formal translation of a traffic light, later to be used in illustrating the meaning of the number 3.

10

Final spread of traffic lights from the booklet "Understanding the Number 3".

11

Initial sketches exploring the triangle as an image for conveying the concept of the number 3.

12

Final layout from the booklet "Understanding the Number 3", using the triangle as the foundation image.

13

Final layout from the booklet "Understanding the Number 3", utilizing the concept of playing cards to illustrate the meaning of the number 3.

9.0 Visualization

9.3 Understanding the Addition and Subtraction of Primary Numbers

14

Initial studies for integrating the numeric designation of cards 1 - 9 with their corresponding line interval to be used in the teaching of addition and subtraction.

15

An extension of the study in 14, limiting the exploration to simple line weight variation.

16

Initial visual studies for integrating the addition sign with the line interval.

17, 18

Final cards used for "Understanding the Addition and Subtraction of Primary Numbers".

19, 20

Four spreads from the binder "Understanding the Addition and Subtraction of Primary Numbers". By flipping the

cards within the binder, a child poses himself an addition or subtraction question and receives the answer on the next page. Color coding was used to indicate the addition or subtraction mode.

9.0 Visualization

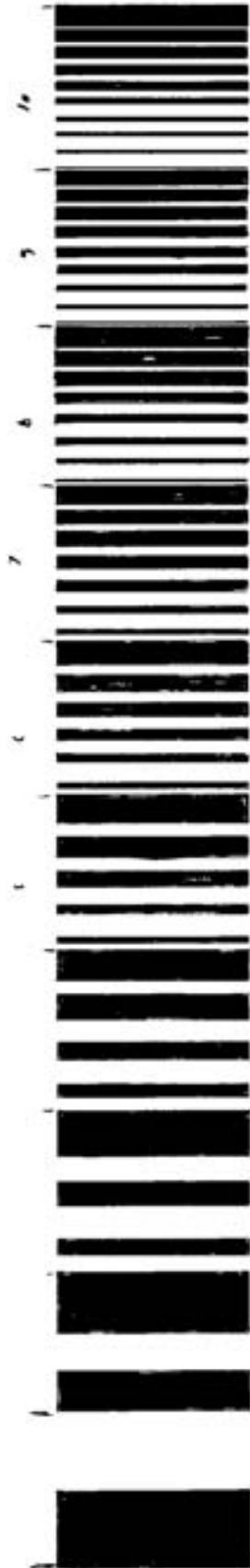
9.4 The Identity

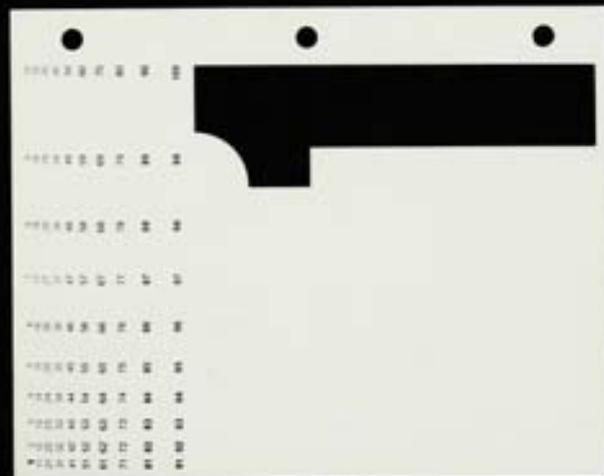
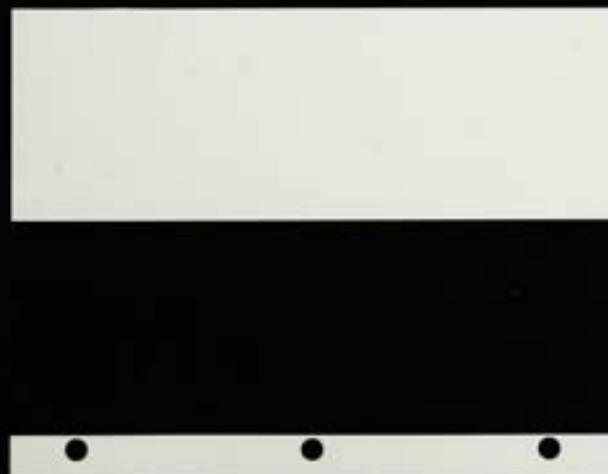
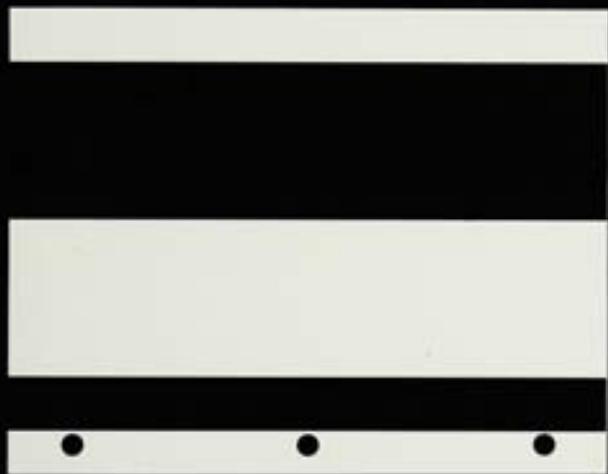
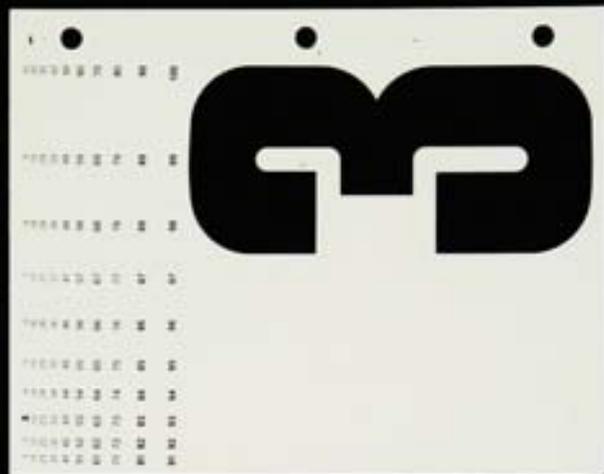
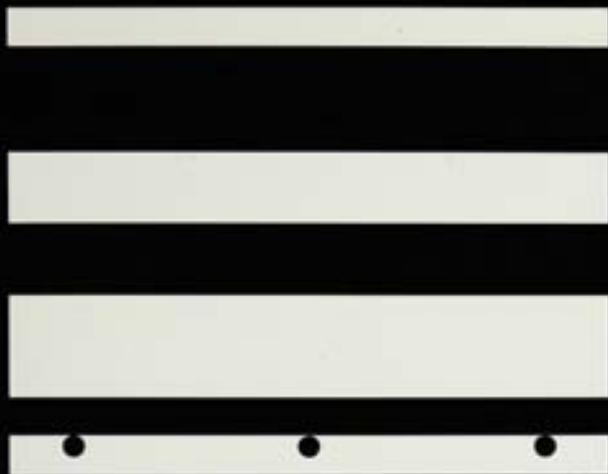
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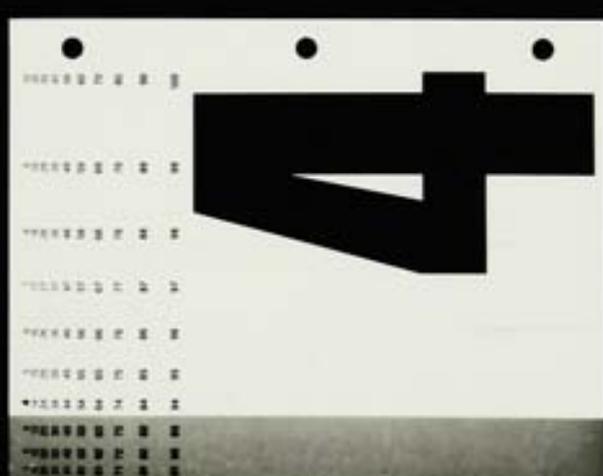
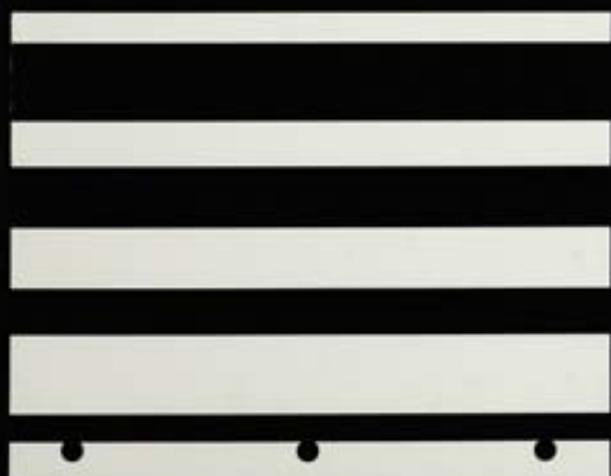
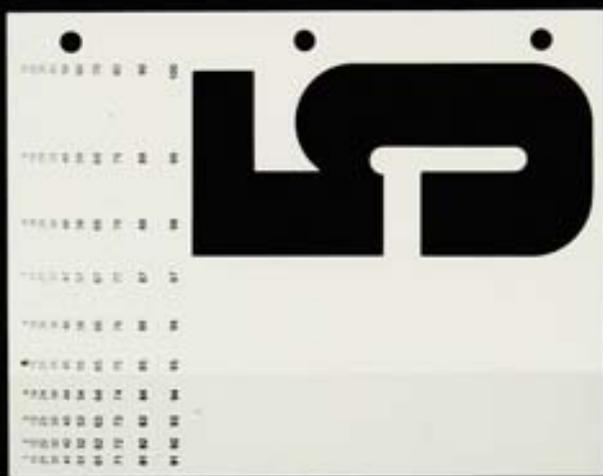
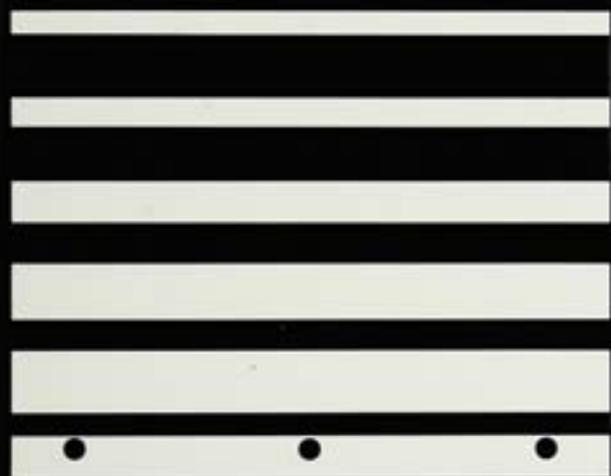
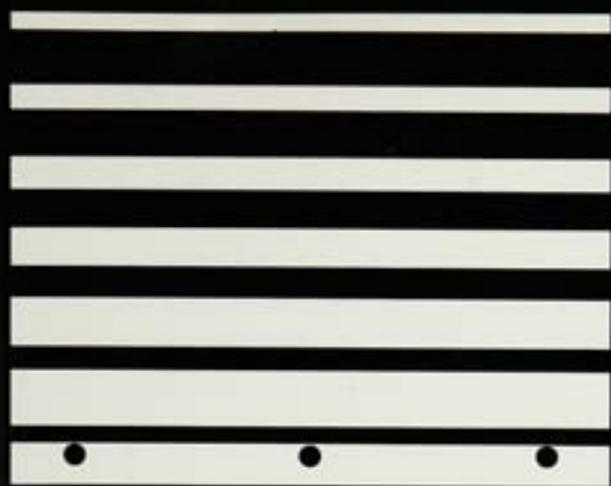
The mini-identity program for CONCEPT 1, showing three components: the two loose-leaf binders, "Understanding the Addition and Subtraction of Primary Numbers" (left), "The Numbers 1 - 100" (right), and the cover of the booklet "Understanding the Number 3".

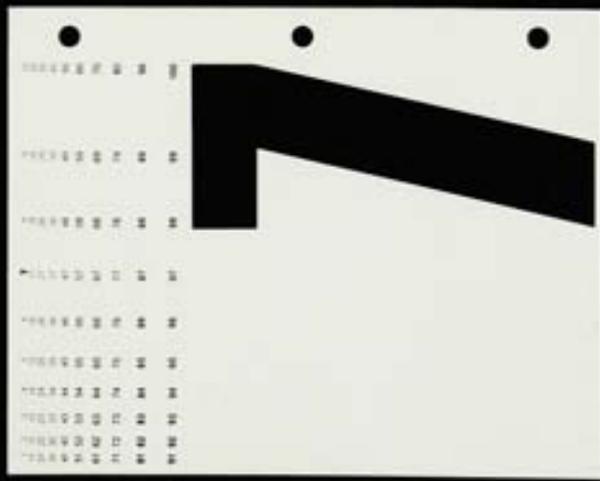
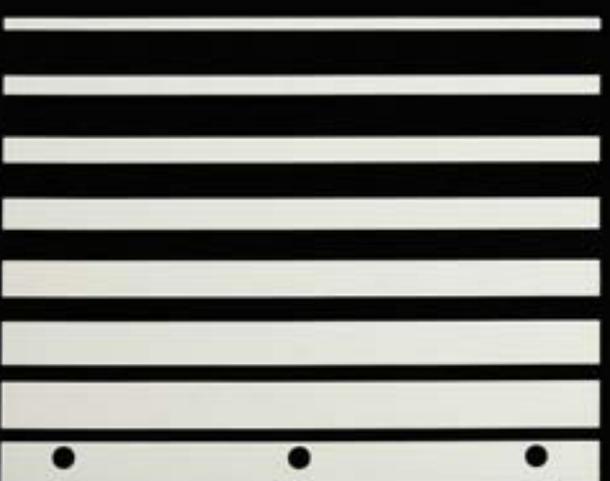
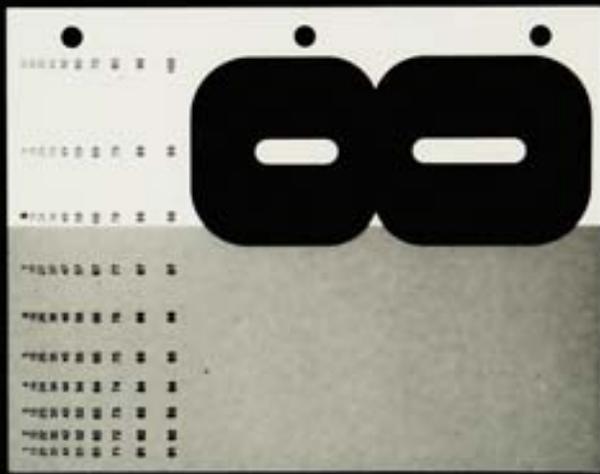
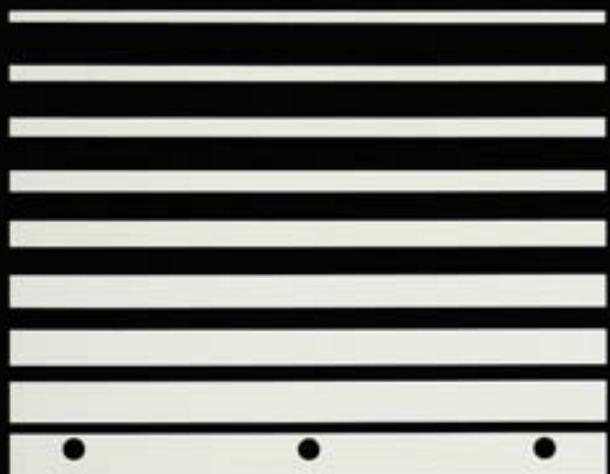
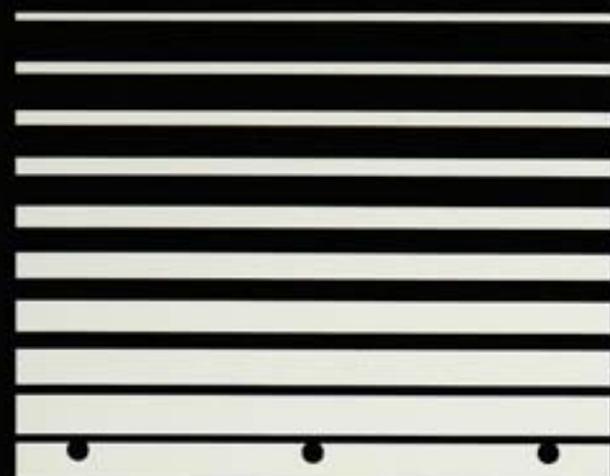
10.0 Bibliography

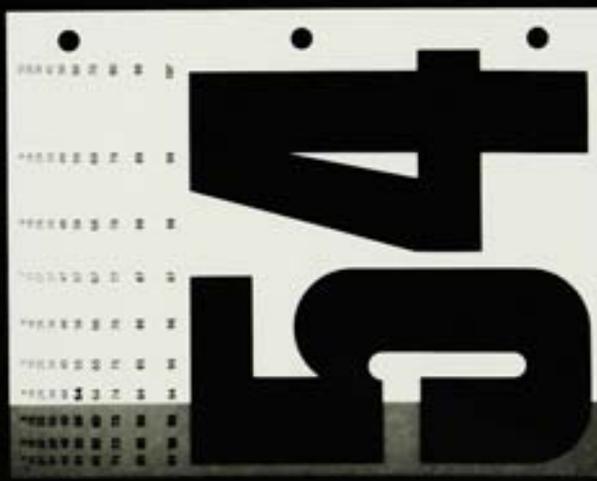
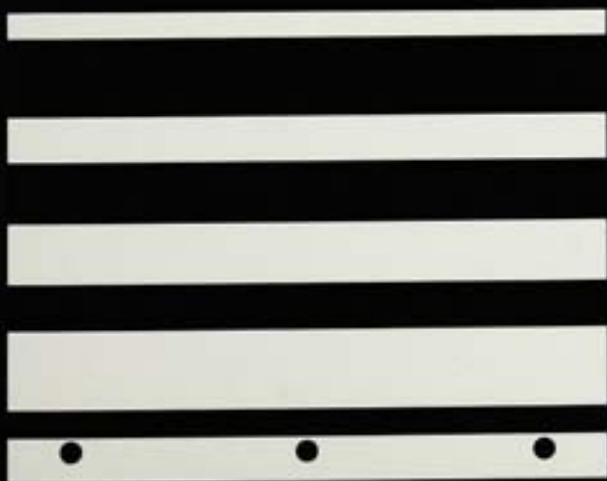
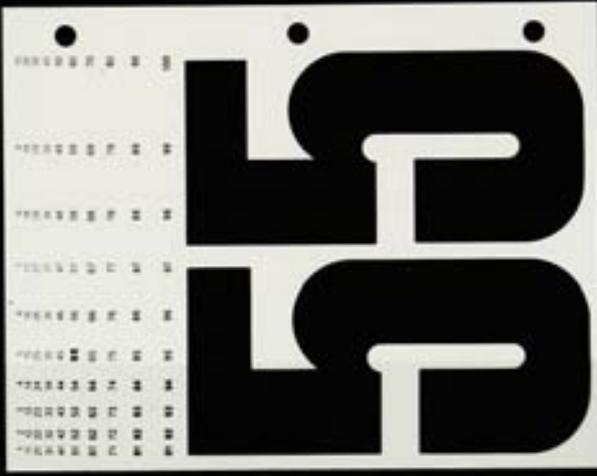
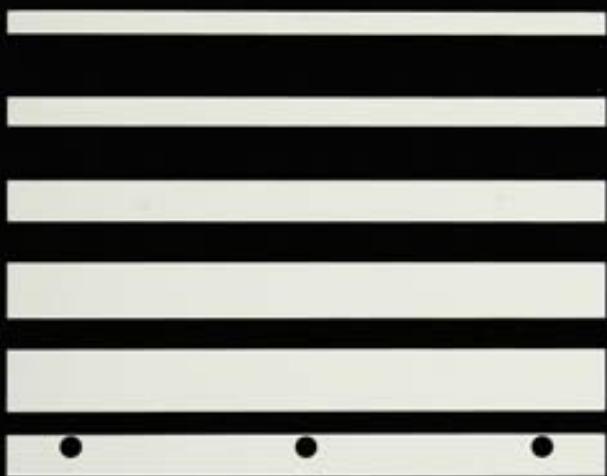
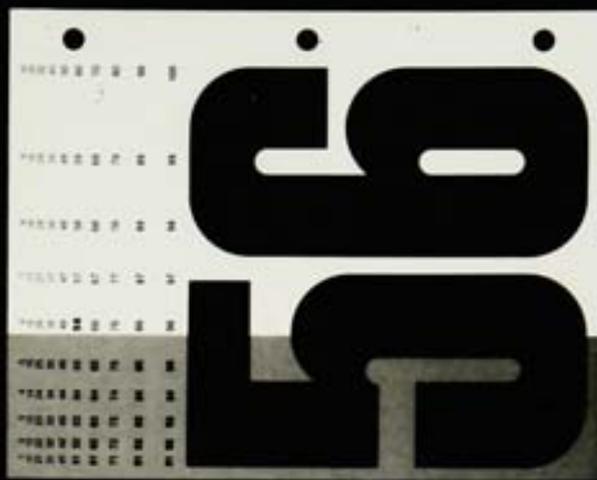
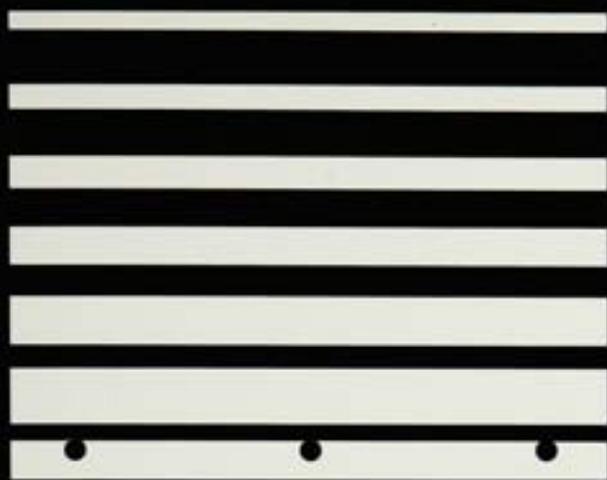
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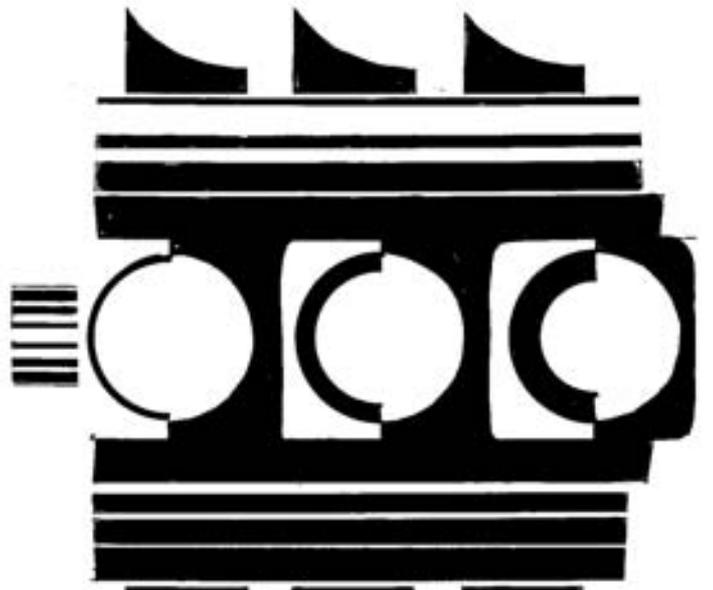
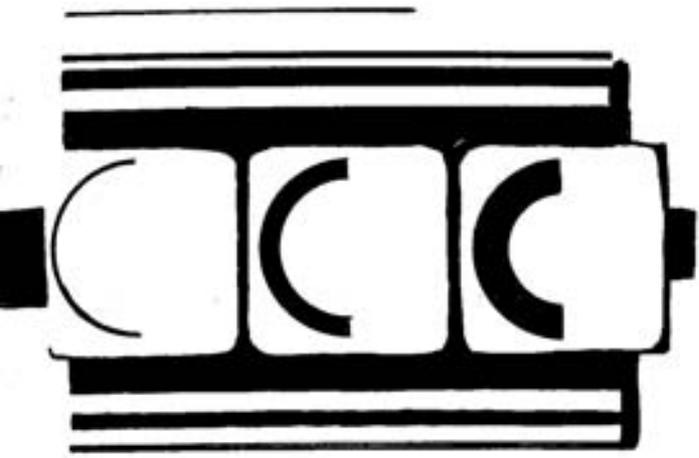




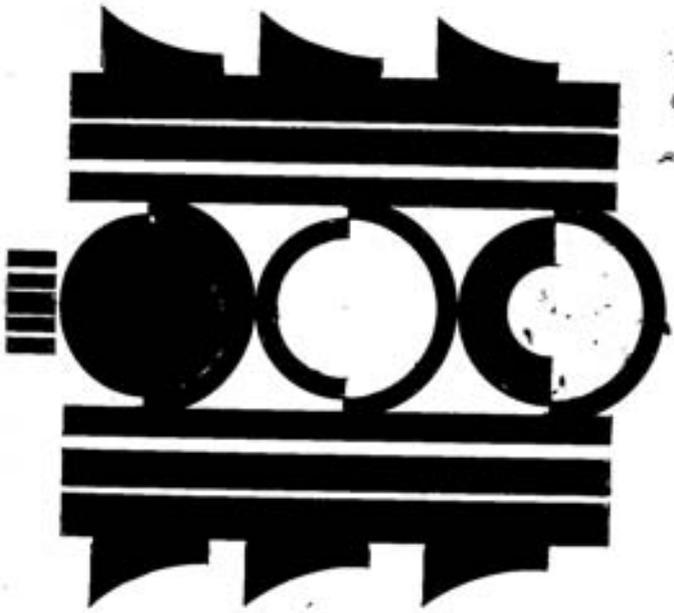


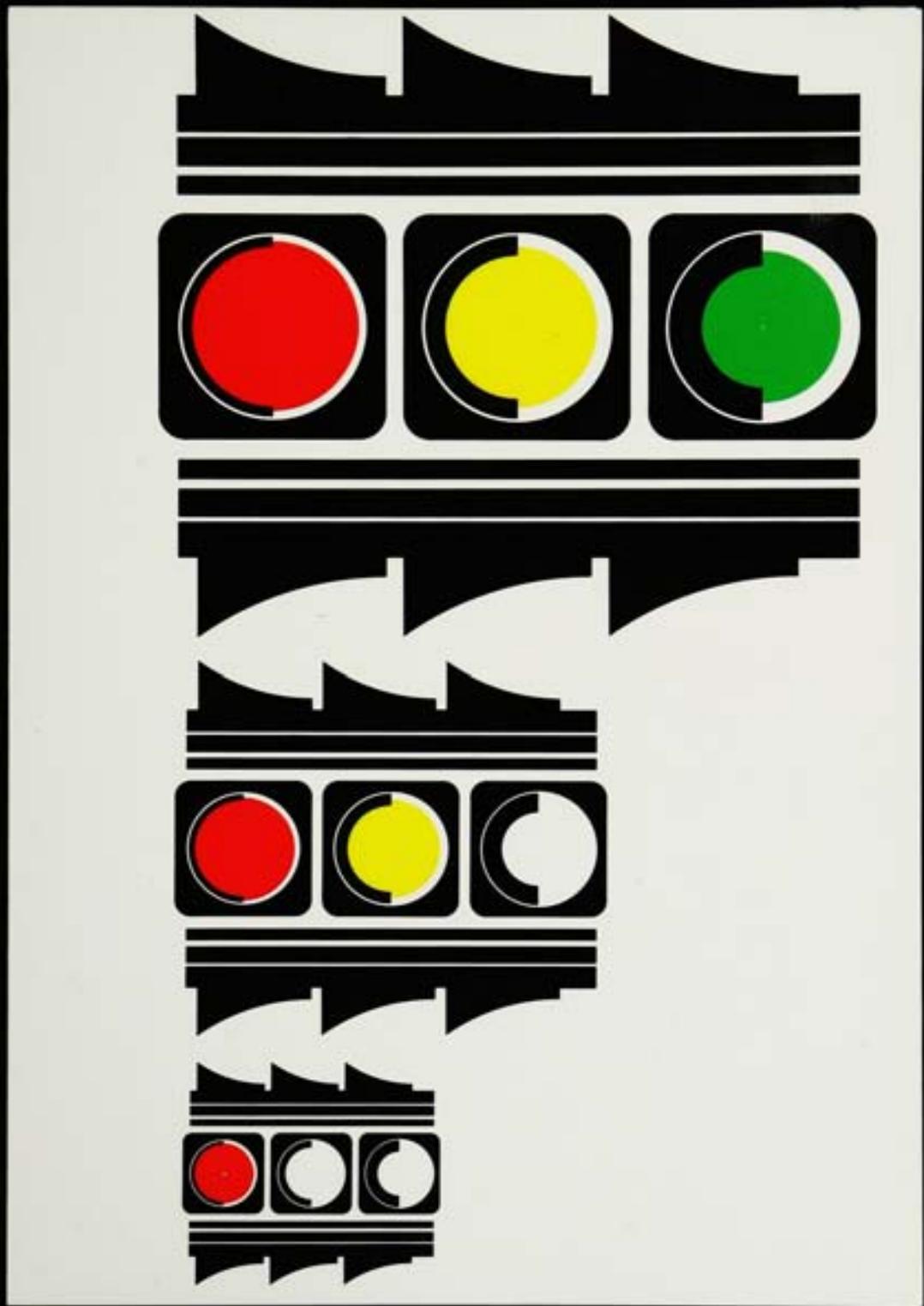


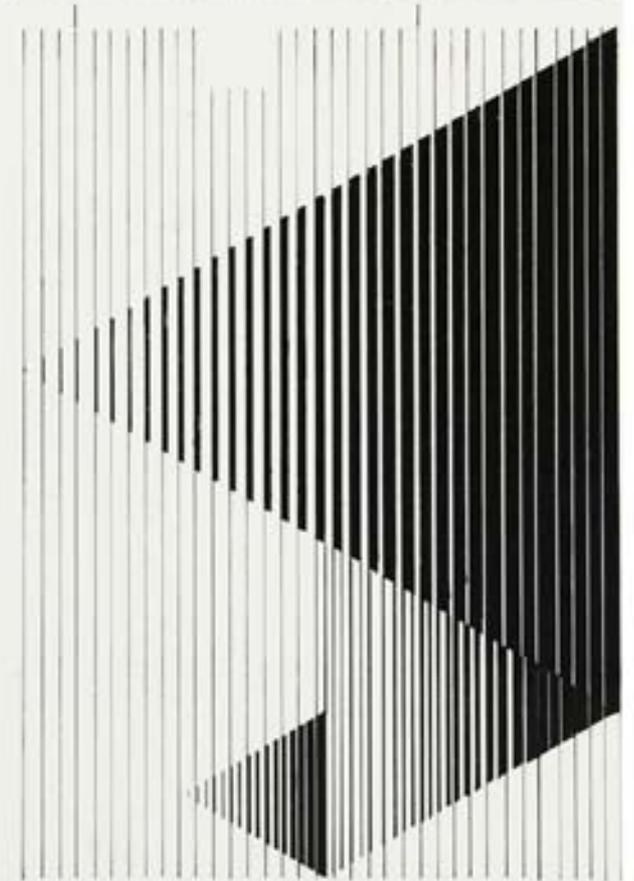
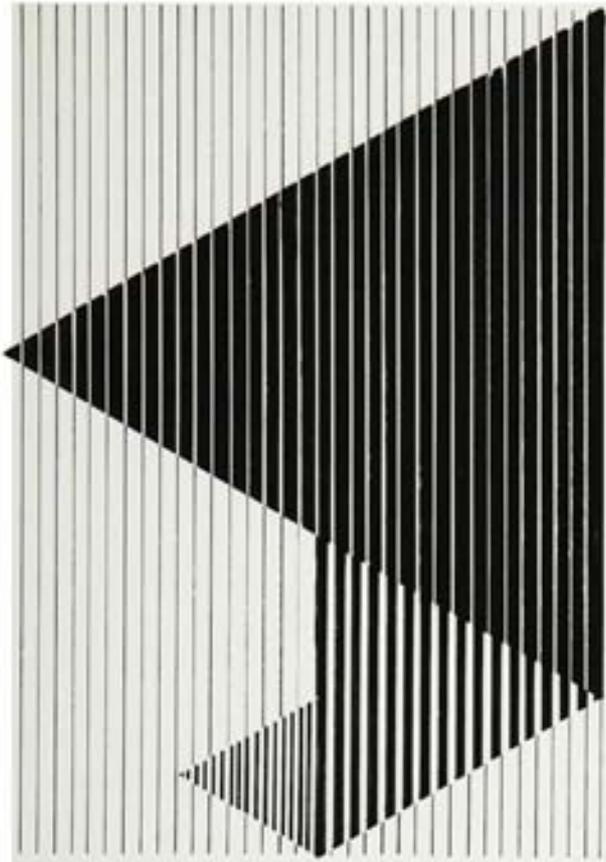




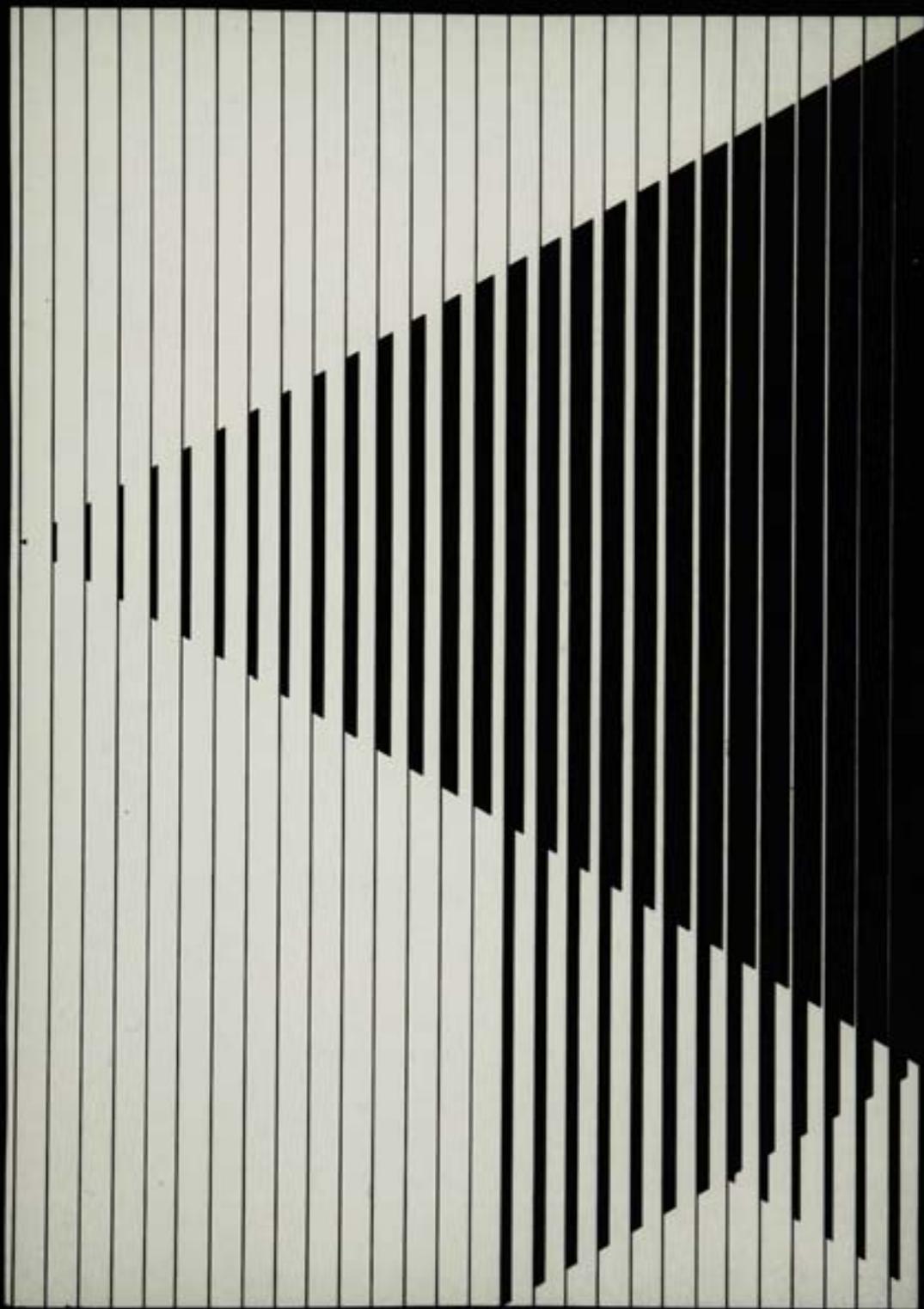
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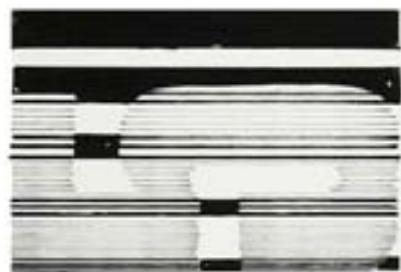
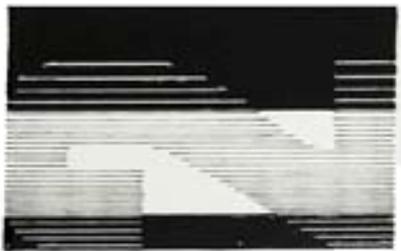
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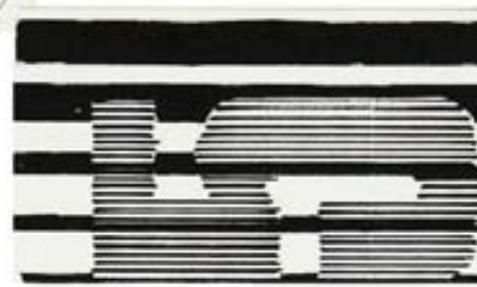
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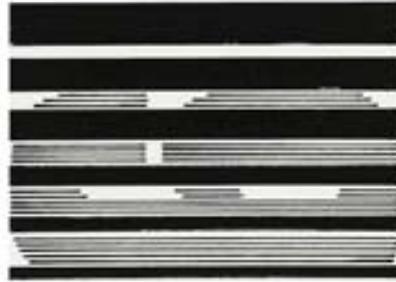
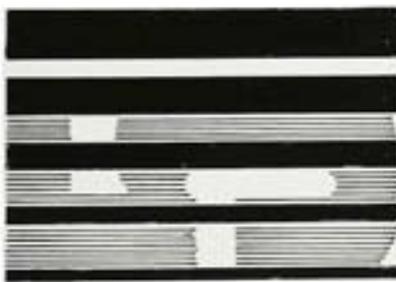
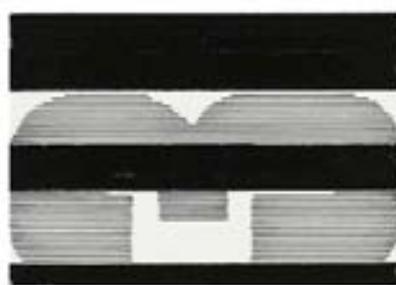
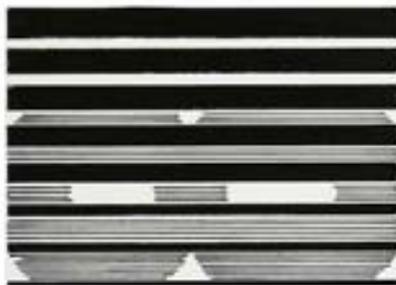
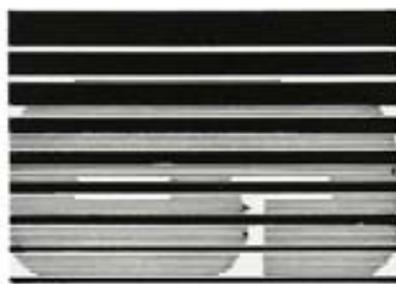
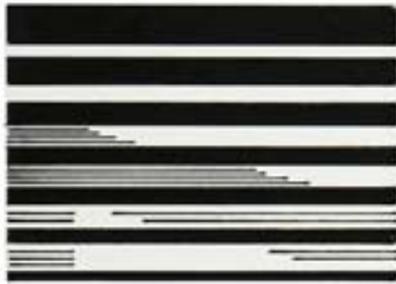
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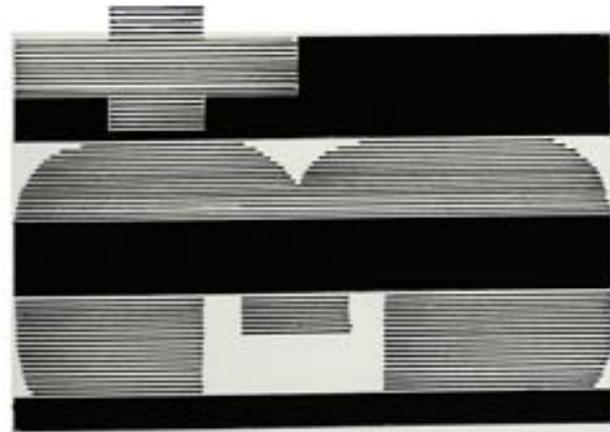
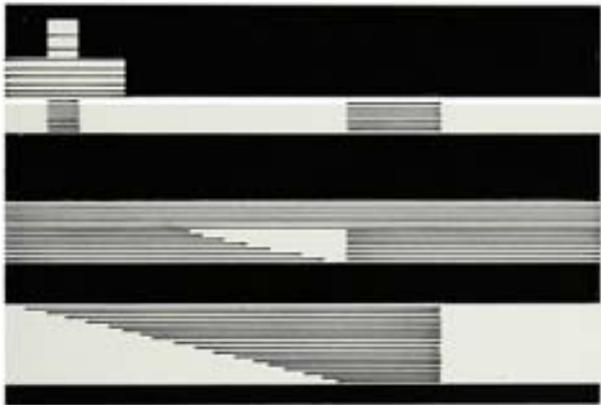
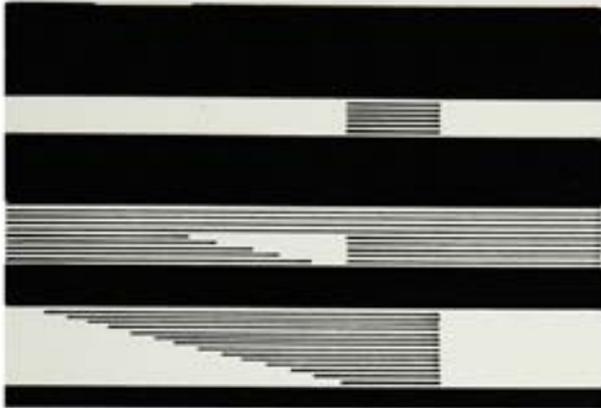


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Line Drawing by M. C. Escher / 1944 - 1945

