Novel method for determining the electromagnetic dispersion relation of periodic slow wave structures

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A novel method for calculating the dispersion relation of electromagnetic modes in an arbitrary periodic slow wave structure is reported. In this method it is sufficient to know the frequencies corresponding to three special wave number values, with other points calculated using an approximate analytical expression. This technique was successfully applied to determine the dispersion relation of the TM_{01} mode in a sinusoidally corrugated slow wave structure. This structure is commonly used in relativistic high-power backward wave oscillators and traveling-wave tubes, and is expected to have many additional applications.

When an electron beam is injected into a periodic slow wave structure, the beam structure resonance often leads to either an absolute or a convective instability. The beam space-charge waves couple to the slow wave structure normal modes to produce microwave and millimeter wave radiation. The group and phase velocity of the slow wave structure modes, as well as the electron beam characteristics, determine the nature and frequency of the beam-wave interaction.

There are many families of microwave and millimeter wave generating devices whose operation depends on this type of interaction. As an example, Fig. 1 shows schematically the regions of operation of some of these devices in the frequency-wave number domain, as well as some typical slow wave structures. Relativistic traveling-wave tubes[1] (RTWTs) operate below the point where the wave number (normalized to the structure period) is equal to π radians. In this case, both the phase and the group velocities are positive. At $\beta L = \pi$, the upper cutoff frequency, the electromagnetic wave undergoes a phase shift of π radians per period of the slow wave structure. At this point, the phase velocity is positive and the group velocity is zero. Backward wave oscillators (BWOs) as well as carcinotrons operate in the region $\pi < \beta L < 2\pi$, where the phase velocity (ω/β) is positive and the group velocity $(\partial\omega/\partial\beta)$ is negative (ω is the angular frequency and β is the wave number). Extended interaction oscillators (EIO) operate very close to the $\beta L = 2\pi$ point.

Relativistic backward wave osillators[2] and related devices which typically operate close to the $\beta L = \pi$ point, prove to be efficient and powerful microwave and millimeter wave sources with reported record power levels reaching 15 GW at wavelength of 3 cm[3] and 5 GW at 3 mm,[4] with efficiencies of up to 50%. In these devices, a smoothly corrugated waveguide has been found to be advantageous over alternative periodic structures due to its high-power handling capabilities. Many other devices (to be discussed later) also employ corrugated waveguide slow wave structures operating in both TM and TE modes. It is important, therefore, to know the dispersion relation of such slow wave structures in order to synchronize the phase velocity of the wave with the electron beam to produce efficient interaction. In this letter we present a novel, accurate, and simple method for calculating the dispersion curve of an infinitely long slow wave structure of arbitrary geometry. This method was successfully applied to determine the dispersion curve of the TM_{01} mode in a sinusoidally corrugated waveguide. The results are then compared with those achieved by using other techniques.

Any periodic slow wave structure with n periods, when shorted at both ends, will exhibit $(n+1)$ resonant frequencies with a phase shift per period equally spaced between 0 and π. Outside of the region $0 \leqslant \beta L \leqslant \pi$, the dispersion relation is periodic in β space (Floquet's theorem). This dispersion relation can be either be calculated using computational methods or measured experimentally (cold test).

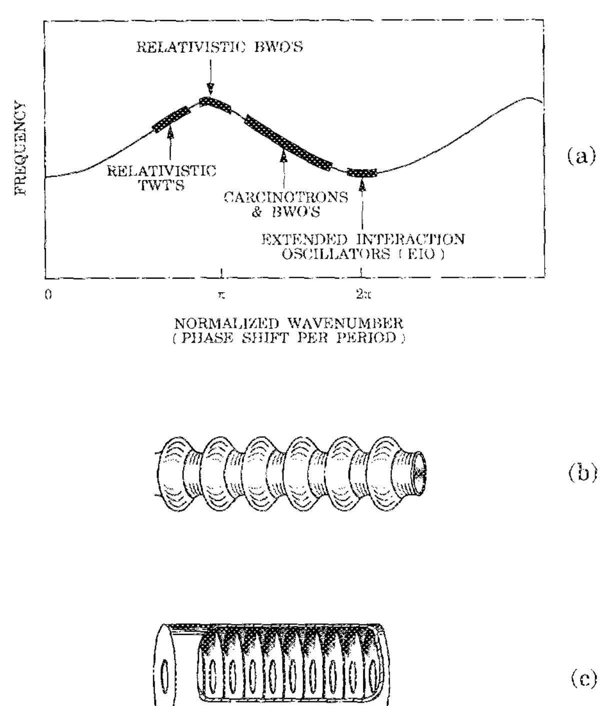

FIG. 1. (a) Regions of operation in the frequency-wavelength domain of various microwave and millimeter wave generators; (b) schematic diagram of a sinusoidally corrugated waveguide; (c) iris loaded waveguide.

TABLE I. Resonance frequencies (GHz) of a sinusoidally corrugated waveguide cavity ($R_0 = 1.5$ cm, $L = 1.67$ cm, $h = 0.273$ cm).

Resonance frequency (GHz)	2-period cavity	4-period cavity
$f_0 = f(\beta L = 0)$	7.404	7.412
$f(\beta L = \pi/4)$	N/A	7.589
$f_{\pi/2} = f(\beta L = \pi/2)$	8.046	8.046
$f(\beta L = 3\pi/4)$	N/A	8.557
$f_\pi = f(\beta L = \pi)$	8.773	8.754

Instead of using a large number of periods to accurately calculate the dispersion relation, we will show that it is sufficient to calculate the short circuit cavity resonant frequencies corresponding to three special normalized wave number values, $\beta L = 0$, $\pi/2$, and π radians by rigorously solving the boundary value problem. With these three frequencies, one can use an analytical expression to produce the complete dispersion curve. Furthermore, we can also analytically calculate the phase and group velocities at any point along the dispersion relation. We have used a two-dimensional electromagnetic code[5] ("Superfish") for the calculation of these three special frequencies for a sinusoidally corrugated waveguide, oriented in the z direction whose radius is described by

$$R = R_0[1 + h\cos(2\pi z/L)], \quad (1)$$

where R_0 is the average radius, h is the normalized corrugation amplitude, and L is the structure period.

For high-power relativistic BWOs, the main mode of interest is the cylindrically symmetric TM_{01} mode. In our test case, the waveguide average radius was $R_0 = 1.50$ cm, the ripple period was $L = 1.67$ cm, and the normalized corrugation amplitude was $h = 0.273$ cm. Superfish modeling yielded (with proper boundary conditions) the three special resonance frequencies in the lower passband. The results are tabulated in the second column of Table I (two period cavity). The corresponding mode patterns (electric field lines) are shown in Figs. 2(a), 2(b), and 2(c), which pertain to phase shifts per period of 0, $\pi/2$, and π, respectively.

Next, the expression used to calculate the complete dispersion relation is discussed. By using the impedance or *ABCD* matrix of a four-terminal network, together with Floquet's theorem[6] to describe a periodic slow wave structure, the dispersion relation can be expressed in the non-explicit form

$$f = f(\cos\beta L, G) \quad (2)$$

where f is the frequency of the electromagnetic radiation and G is a geometric factor related to a specific slow wave structure dimension. Let $A = f_{\pi/2}$, $B = (f_\pi - f_0)/2$, $C = f_{\pi/2} - (f_\pi + f_0)/2$. It can be shown that the dispersion relation of any general periodic structure can be expressed in the following explicit, approximate form:

$$f = A - B\cos\beta L - C\cos^2\beta L. \quad (3)$$

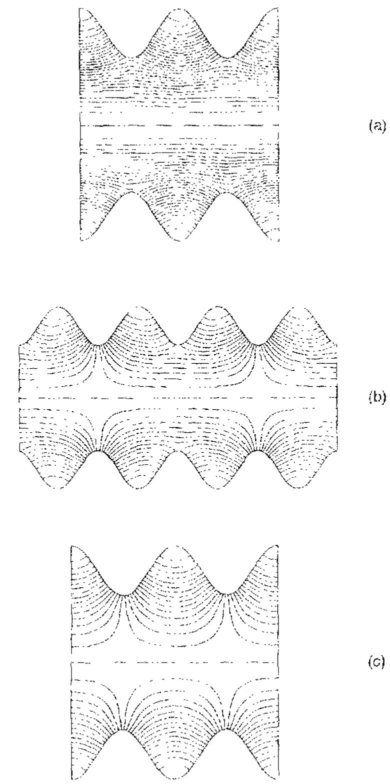

FIG. 2. (a) Mode patterns (electric field lines) corresponding to zero phase shift, (b) $\pi/2$ phase shift, and (c) π phase shift in the TM_{01} lower pass band of a sinusoidally corrugated waveguide.

The three terms in Eq. (3) are sufficient to ensure excellent accuracy for most practical cases, as will be shown later. Notice that the geometrical factor G (as yet unspecified) does not appear in the equation. Rather, it enters indirectly through A, B, C, which are geometry-dependent parameters with simple physical meaning. The first term, A, represents the value of the frequency near the midband. The second is an increment term whose maximum value equals half the difference between the upper and lower cutoff frequencies of the passband. The third term is a correction term. The combination of all three terms exactly satisfied the dispersion relation at the three special wave numbers (0, $\pi/2$, and π) and is an excellent approximation at all other wave numbers, as will be shown later.

Equation (3) enables one to directly calculate the phase and group velocities at any point along the dispersion relation. The expressions derived for the velocities are given in Eqs. (4) and (5):

$$v_{\text{phase}} = \frac{[2L\pi f_{\pi/2}(1 + \Delta f/f_{\pi/2})]}{\cos^{-1}\{[(B^2 - 4C\Delta f)^{1/2} - B]/2C\}}, \quad (4)$$

$$v_{\text{group}} = \partial\omega/\partial\beta = 2\pi L(B\sin\beta L + 2C\cos\beta L\sin\beta L), \quad (5)$$

where $\Delta f = f - f_{\pi/2}$.

The results of our test case are plotted in Fig. 3. It shows the TM_{01} dispersion curve in the lower passband for the sinusoidally corrugated waveguide Fig. 3(a), as well as that of an iris loaded waveguide having the same minimum and maximum radial dimensions. The two are similar with

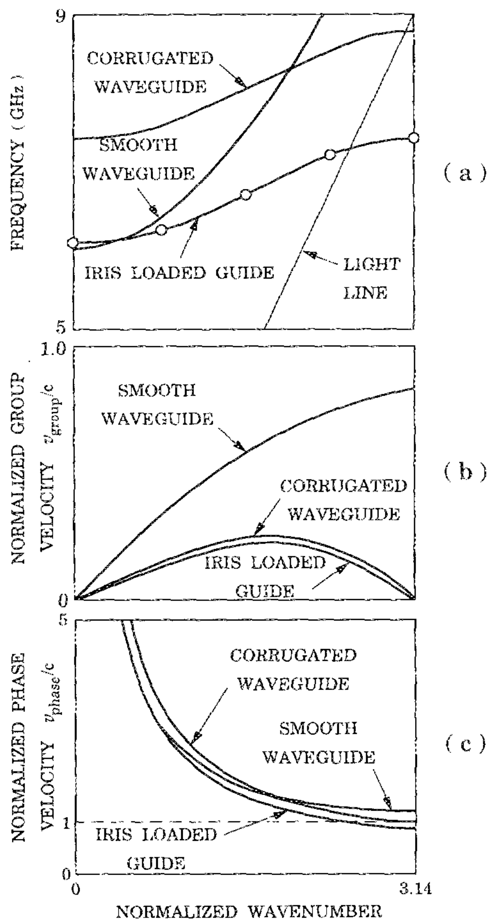

FIG. 3. (a) TM_{01} dispersion curve for a sinusoidally corrugated, iris loaded, and smooth waveguide, (b) normalized group velocities, (c) phase velocities.

the iris loaded guide dispersion curve always lower in frequency compared to the corrugated guide. This is expected since the volume of the first is larger than that of the second, driving all the resonant frequencies down. The open circles at $\beta L = 0$, $\pi/2$, and π were calculated by using Superfish and were used in Eq. (3) to calculate the complete dispersion relation. For comparison, the dispersion relation of a smooth waveguide is also given. Figures 3(b) and 3(c) display the phase and group velocities (normalized to the speed of light) for the same cases [Eqs. (4) and (5)].

In order to estimate the accuracy of this technique as applied to the sinusoidally corrugated guide, we first calculate additional resonance points on the dispersion curve, using "Superfish". Four periods of the corrugated guide cavity were modeled, yielding five resonances in the wave number range of 0 to π and the results are given in the third column of Table I (four-period cavity) and are plotted as additional open circles in Fig. 3(a) with very good agreement (eight periods were also successfully modeled). As an independent check, the dispersion relation derived here was compared to the dispersion relation derived elsewhere[7,8] for the same corrugated guide geometry by solving a system of coupled linear differential equations. Figure 4 shows a comparsion between the dispersion curve

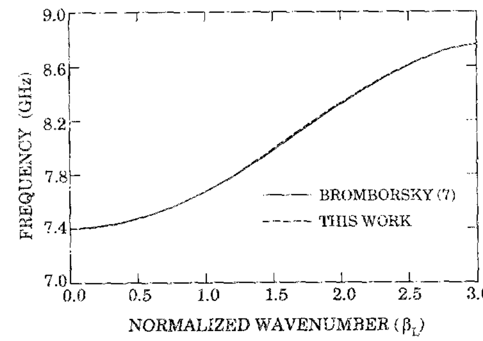

FIG. 4. Comparison of the TM_{01} dispersion curve of Ref. 7 (solid line) and this work (dotted line) for a sinusoidally corrugated waveguide.

as calculated by these two techniques [Ref. 7 and Eq. (3)] for the same corrugated guide. The results are in excellent agreement (within 0.15%) over the entire wave number range $0 < \beta L < \pi$. Outside this range the dispersion relation is, of course, periodic.

It is expected that this technique will be beneficial for determining the dispersion curve of periodic structures of arbitrary shapes. The three special resonance frequencies which are needed in order to generate the complete, general dispersion curve can be calculated by rigorously solving the boundary problem or measured experimentally.

Corrugated slow wave structures may prove beneficial in prospective applications such as TE mode slow wave cyclotron amplifiers,[9] TM mode relativistic extended interaction oscillators and amplifiers,[10] TE mode CARMs,[11] electromagnetically pumped free-electron lasers,[12,13] and plasma-loaded backward wave oscillators.[14]

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