A Novel Highly Accurate Synthetic Technique for Determination of the Dispersive Characteristics in Periodic Slow Wave Circuits

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***Abstract*—A novel, highly accurate (0.1–0.5%) synthetic technique for determining the complete dispersive characteristics of electromagnetic modes in a spatially periodic structure is presented. It was successfully applied for the cases of the fundamental ($TM_{0(1)}$) as well as higher order ($TM_{0(2)}$, $TM_{0(3)}$) passband modes in a corrugated waveguide. This structure is commonly used in relativistic backward wave oscillators, traveling wave tubes, extended interaction oscillators and a variety of multiwave Cerenkov generators. An appropriately shorted periodic structure resonates at specific frequencies. To accurately and unambiguously measure these frequencies we used unique antenna radiators to excite pure modes in the circuit under test. An analytical technique to derive the complete dispersion relation using the experimentally measured resonances is presented. This technique, which is based on the intrinsic characteristics of spatially periodic structures, is applicable to slow wave structures of arbitrary geometry.**

I. Introduction

When an electron beam is injected into a periodic slow wave structure, the beam wave coupling often leads to either an absolute or a convective instability. The group and phase velocities of the slow waves in the structure and the electron beam characteristics determine the nature and frequency of the beam-wave interaction.

There are many families of microwave and millimeter wave generating devices, both relativistic and nonrelativistic, whose operation depends on the location of the interaction region along a dispersion curve. Fig. 1 shows schematically the regions of operation of various devices in the frequency-wave number domain. Relativistic traveling-wave tubes [1] operate below the point where the normalized wave number, βL, is equal to π radians where L is the structure period. In this case, both the phase and the group velocities are positive. At $\beta L = \pi$, the upper cutoff frequency, the electromagnetic wave undergoes a phase shift of π radians per period of the slow wave structure. At this point, the phase velocity is positive and the group velocity is zero. Backward wave oscillators [2] (BWOs) as well as carcinotrons and multiwave Cerenkov generators [3] operate in the region $\pi < \beta L < 2\pi$, where the phase velocity (ω/β) is positive and the group velocity ($\partial\omega/\partial\beta$) is negative ($\omega$ is the angular frequency and β is the wave number). Extended interaction oscillators (EIO) operate very close to the point where $\beta L = 2\pi, 3\pi, \cdots$ [4]. Despite the fact that the operational mode of these devices is usually the lowest passband symmetric transverse magnetic ($TM_{0(1)}$) mode, some devices such as multiwave diffraction generators [3] operate at higher passband transverse modes ($TM_{0(n)}$), $n, > 1$), as can be seen from Fig. 1.

Relativistic backward wave oscillators [2] and related

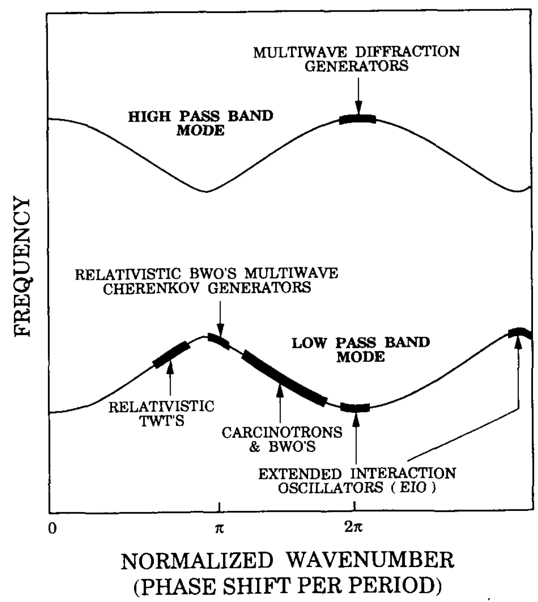

Fig. 1. Regions of operation in the frequency wave number domain of various microwave and millimeter wave sources.

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devices have proven to be efficient and powerful microwave and millimeter wave sources with reported record power levels reaching 15 GW [3] at wavelength of 3 cm and 5 GW at 3 mm, [5] with efficiencies of up to 50%. It is important, therefore, to know the dispersion relation both for the lowest and higher passband modes of such slow wave structures in order to synchronize the phase velocity of the wave with the electron beam to produce efficient interaction.

In principle the dispersive characteristics of arbitrary slow wave circuits can be determined point by point using numerical techniques or by performing cold tests of the circuits themselves. Numerical two-dimensional (2-D) RF codes [6] are usually restricted to lower passband modes and cannot be easily used for analyzing high passband modes or overmoded systems (in which the slow wave circuit diameter is much greater than wavelength). Other numerical codes [7]–[9] which can calculate higher order passband modes are structure dependent and require code modifications each time the structure geometry is changed.

The experimental approach is therefore of considerable importance not only for final verification of the actual slow wave circuit dispersive characteristics but also for checking the validity of the computer code which has been employed for the theoretical design of the circuit. For this reason its accuracy is of vital importance. At present, most of the studies on slow wave circuits are based on analysis and numerical calculation because of the difficulty in accurately measuring the complete dispersion curve experimentally. It is well known that any periodic slow wave structure of N periods, when shorted appropriately at both ends, will exhibit $(N + 1)$ discrete resonant frequencies with phase shifts per period equally spaced between 0 and π. Generally speaking, these specific resonant modes can be determined experimentally and then used to fit to a curve to give the complete dispersion function. However, it is very difficult to accurately implement the above procedure because of problems associated with field distortion due to the finite size of the mode launching probe, spurious resonances induced by imperfect input excitation devices and the limited accuracy of curve fitting techniques due to the limited number of data points obtained by the experimental apparatus.

In this paper we present a novel, synthetic technique to accurately (0.1–0.5%) determine the dispersive characteristics of periodic slow wave structures of arbitrary geometry. This method was successfully applied to determine the complete dispersion curves of the lowest ($TM_{0(1)}$) as well as higher ($TM_{0(2)}$ and $TM_{0(3)}$) order passband modes in a sinusoidally corrugated waveguide which has been extensively used for higher power microwave generation.

The principles of the measurement, the experimental results, and a brief description of the launchers are given in Section II. Determination of the complete dispersion relation (interpolation between points) is based on the intrinsic characteristics of periodic slow wave structures—rather than on an equivalent circuit model [10] or an arbitrary interpolation. This topic is covered in Section III. Section IV is devoted to the excitation of pure modes in periodic waveguides, followed by a discussion and summary (Section V).

We wish to clarify the notation adopted here to describe the type of wave propagation in periodic structures. The axisymmetric transverse magnetic modes in spatially periodic structures will be referred to in this work as $TM_{0(n)}$ *passband modes*, in analogy to the notation used for wave propagation in smooth waveguide ($TM_{o(n)}$). However, there is a big difference between the two: Here (n) is the index number of the passband and does not correspond to the number of radial field variations of fast waves in smooth waveguides. In this way $TM_{0(1)}$ will refer to the lowest frequency passband, $TM_{0(2)}$ to the next one up and so on for higher and higher frequencies (see Figs. 1 and 5).

II. Description and Results of the Experiment for Determination of Specified Points on the Dispersion Curve

Accurate determination of specified points on the dispersion of periodic slow wave structures by experiment is usually difficult and is mostly restricted to the fundamental mode of the slow wave circuit. As will be seen from our experimental results, those problems were solved due to the successful use of two novel mode launchers.

We have designed, constructed and tested a modular X-band slow wave structure in the form of a rippled wall waveguide having six periods, as shown in Fig. 2 (for dimensions, see (7)). When the circuit is shorted appropriately at both ends, and excited by cage or rod-wheel mode launchers (Fig. 8), the cavity accurately resonates at seven specific frequencies corresponding to wavenumbers with phase shift per period of 0, $\pi/6$, $2\pi/6$, $3\pi/6$, $4\pi/6$, $5\pi/6$, and π. Generally, the resonant approach is attractive for measuring the dispersion characteristics of a properly shorted slow wave circuit because of its potentially high accuracy, but the difficulty usually lies in choosing the most suitable excitation launcher. Single axial pins, axial loops and side coupling loops [11] were all judged to be inadequate from a mode purity point of view. Obviously, single loops cannot excite symmetric modes. Single on-axis probes cannot effectively excite slow waves for which the on-axis field should not be maximum, [12] nor can they provide sufficient coupling to the cavity under test so as to distinguish resonance absorption from nonresonance reflection. It is therefore almost impossible to measure the dispersive characteristics of high order passband modes with conventional probes. In this work we used two new types of input excitation devices. The first is a cage antenna radiator (Fig. 8(b)) and the second is a rod-wheel radiator (Fig. 8(a)). Both yielded excellent experimental results. A more complete description of these launchers will be given in Section V.

Figure 3 shows the experimentally measured resonant absorption peaks corresponding to seven axial modes of

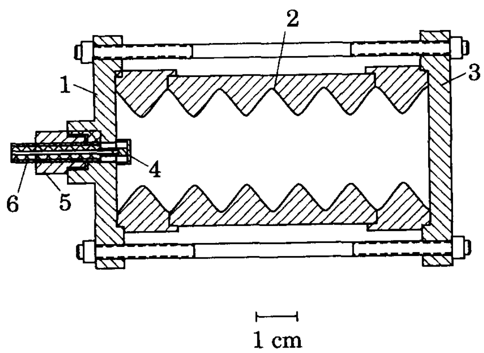


Fig. 2. A modular X-band slow wave structure in the form of rippled waveguide used for measuring the special resonance frequencies; (1,3) shorting planes, (2) corrugated wall slow wave circuit, (4) cage antenna, (5,6) RF feed and support.

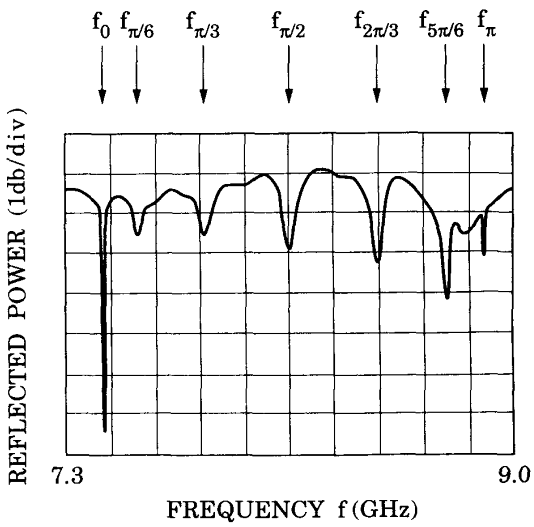


Fig. 3. The specific resonances corresponding to seven axial modes of the corrugated wall cavity associated with the fundamental, $TM_{0(1)}$ passband mode.

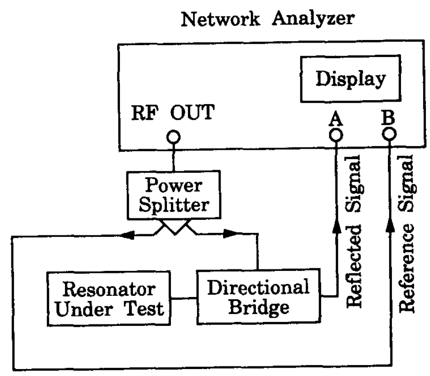


Fig. 4. Block diagram of the system used for measuring the dispersion relation of a slow wave structure by means of resonant approach.

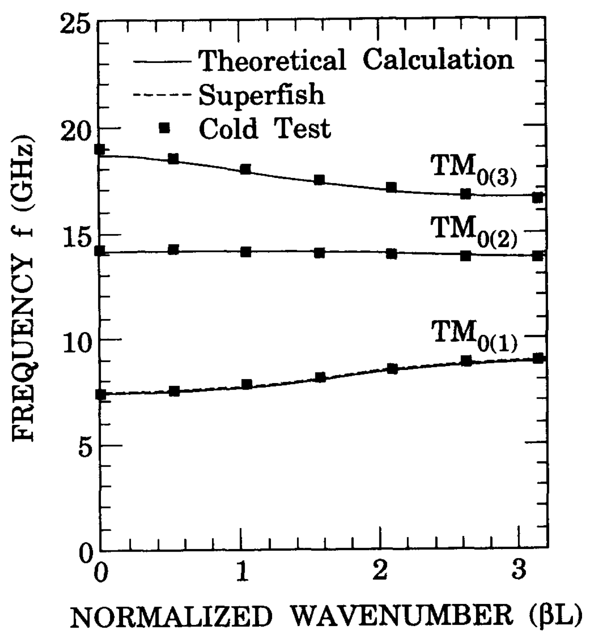
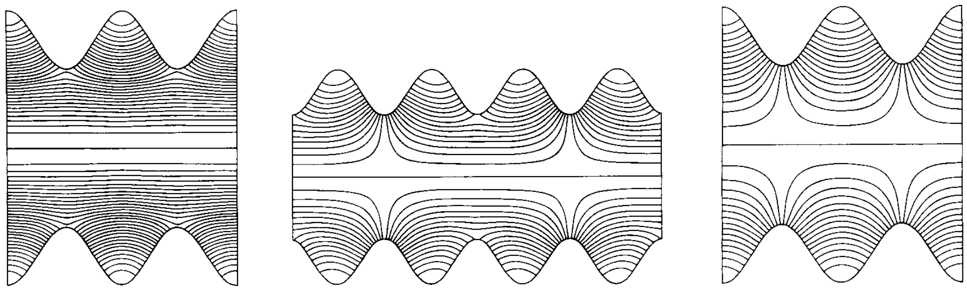


Fig. 5. The dispersion diagram of the sinusoidally corrugated waveguide $R_0 = 1.67$ cm, $\delta = 0.273$, $d = 1.67$ cm.

the cavity associated with the $TM_{0(1)}$ passband mode using the rod-wheel launcher. The resonances were measured by looking at the reflection coefficient of the slow wave structure over the appropriate frequency range (ranging from below the lower cutoff to above the upper cutoff of the $TM_{0(1)}$ passband mode). This was achieved with a network analyzer system as shown in Fig. 4. Using the same experimental setup and the same mode launcher, 14 additional resonant peaks associated with $TM_{0(2)}$ and $TM_{0(3)}$ modes were measured with a high degree of accuracy. These resonant peaks are all marked as black squares in Fig. 5. Fig. 6 gives the electric field structure corresponding to three of the seven absorption peaks: zero phase shift (6a), $\pi/2$ phase shift (6b), and π phase shift (6c) in the passband of the lowest order symmetric $TM_{0(1)}$ mode, as calculated by a 2-D RF code (Superfish) [6].

Table I summarizes the measured resonance frequencies as well as those calculated by various numerical techniques for our six period rippled wall cavity.



(a) (b) (c)

Fig. 6. Calculated mode patterns (electric field lines) corresponding to (a) zero phase shift, (b) $\pi/2$ phase shift, and (c) π phase shift in the $TM_{0(1)}$ lower pass band of the sinusoidally corrugated waveguide used in this work.

TABLE I
COMPARISON BETWEEN THE MEASURED AND CALCULATED RESONANCE ABSORPTION FREQUENCIES OF A SIX-PERIOD SINUSOIDALLY CORRUGATE WALL CAVITY (GHz) (R_0 = 1.5 cm, δ = 0.273, d = 1.67 cm)

Resonant Freq. Symbol	$TM_{0(1)}$				$TM_{0(2)}$			$TM_{0(3)}$		
	Experiment	2-D RF Code[6]	Bromborsky & Ruth[7]	Lou & Carmel[9]	Experiment	Bromborsky & Ruth[7]	Lou & Carmel[9]	Experiment	Bromborsky & Ruth[7]	Lou & Carmel[9]
$f_0 = f_0$	7.44	7.40	7.37	7.40	14.29	14.28	14.22	19.06	18.77	18.70
$f_1 = f_{\pi/6}$	7.57	7.49	7.45	7.48	14.26	14.26	14.21	18.57	18.51	18.46
$f_2 = f_{\pi/3}$	7.83	7.72	7.72	7.70	14.18	14.20	14.17	18.04	17.87	17.92
$f_3 = f_{\pi/2}$	8.15	8.05	8.02	8.02	14.06	14.10	14.09	17.49	17.35	17.35
$f_4 = f_{2\pi/3}$	8.48	8.40	8.38	8.37	13.92	13.94	13.97	17.00	16.87	16.90
$f_5 = f_{5\pi/6}$	8.75	8.67	8.67	8.64	13.81	13.78	13.83	16.67	16.59	16.66
$f_6 = f_{\pi}$	8.88	8.77	8.79	8.75	13.76	13.71	13.76	16.52	16.49	16.60

III. DETERMINATION OF THE COMPLETE DISPERSION RELATION FROM THE MEASURED DISCRETE RESONANCES

The complete dispersion curves of arbitrary slow wave circuits can be obtained, in principle, by measuring very large numbers of discrete resonances associated with long structures (large number of periods, $N >> 1$). Since this is not practical, one needs some form of interpolation between small number of experimentally measured points on the dispersion relation. One possibility is to use an interpolation function which is based on an equivalent lumped element circuit [10]. With this technique, however, the results of the interpolation depend on the specific equivalent circuit used, and high accuracy cannot be ensured. In this section we shall show that it is sufficient to know a few resonances (as few as three in many cases) in order to derive a direct analytical expression to produce the complete dispersion curve. Furthermore, we can analytically calculate the phase and group velocity at any point along the dispersion relation. This technique, which is derived from first principles, takes into account the *intrinsic characteristics* of periodic slow wave circuits and can be successfully applied to the fundamental as well as to high order symmetric TM modes.

We now discuss some aspects of the expression used to calculate the complete dispersion relation. By using the impedance or ABCD matrix of a four-terminal network, together with Floquet's theorem to describe a periodic slow wave structure, the dispersion relation can be expressed in the flowing nonexplicit form.

$$f = f(\cos \beta L, G) \tag{1}$$

where f is the frequency of the electromagnetic radiation, βL is the phase shift per period, and G is a geometric factor related to a specific slow wave structure dimension. From (1) we see that f is an even periodic function of βL with a period of 2π. Therefore, the dispersion relation can be written in the form of series:

$$f = \sum_{m=0}^{\infty} a_m \cos m\beta L. \tag{2}$$

The explicit expression for G is not needed for our analysis. Notice that G, as yet unspecified, does not appear in (2). Rather, it enters indirectly through $a_0, a_1, a_2, \cdots, a_m$ which are geometry-dependent parameters. These parameters can be analytically determined by measuring (or numerically calculating) some specific resonant frequencies of the slow wave structure which correspond to specific wavenumbers. For example, in order to calculate the geometry dependent parameters $a_0, a_1, a_2, \cdots, a_m$, all we need do is experimentally measure the specific resonance frequencies $f_0, f_1, f_2, \cdots, f_m$ and invert the following matrix equation.

$$\begin{bmatrix} f_0 \\ f_1 \\ f_2 \\ \vdots \\ f_m \end{bmatrix} = \begin{bmatrix} \cos 0 & \cos 0 & \cos 0 & \cdots & \cos 0 \\ \cos 0 & \cos \pi/m & \cos 2\pi/m & \cdots & \cos m\pi/m \\ \cos 0 & \cos 2\pi/m & \cos 4\pi/m & \cdots & \cos 2m\pi/m \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ \cos 0 & \cos m\pi/m & \cos 2m\pi/m & \cdots & \cos m\pi \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \\ a_2 \\ \vdots \\ a_m \end{bmatrix}. \tag{3}$$

Equation (3) can be solved analytically or numerically in order to obtain the unknowns a_m. It will be shown that three terms in (2) are often sufficient to ensure excellent accuracy ($<1\%$) for most practical cases. We shall next discuss two specific examples: obtaining a complete dispersion relation with (a) three ($m = 0, 1, 2$) and (b) seven ($m = 0, 1, 2, \cdots, 6$) terms in (2). In both cases it is straightforward to obtain analytical expressions for the unknown coefficients. For the case (a) we obtain

$$\begin{aligned} a_0 &= \tfrac{1}{4}(f_0 + 2f_{\pi/2} + f_\pi) \\ a_1 &= \tfrac{1}{2}(f_0 - f_\pi) \\ a_2 &= \tfrac{1}{4}(f_0 - 2f_{\pi/2} + f_\pi) \end{aligned} \tag{4}$$

and the complete dispersion relation (2) can be approximated for each mode by

$$f \cong \sum_{m=0}^{m=2} a_m \cos m\beta L. \tag{5}$$

It can be shown that the dispersion relation can also be expressed for this case in a slightly different form:

$$f = A - B \cos \beta L - C \cos^2 \beta L \tag{6}$$

where $A = f_{\pi/2}$, $B = (f_\pi, - f_0)/2$, $C = f_{\pi/2} - (f_\pi + f_0)/2$. The first term ($A$) in (6) represents the value of the frequency near the midband. The second term (B) is an increment term whose maximum value equals half the difference between the upper and lower cutoff frequencies of the passband. The third term (C) is a correction term. The combination of all three terms exactly satisfies the dispersion relation at the three special wavenumbers ($\beta L = 0, \pi/2, \pi$) and is an excellent approximation at all other wave numbers.

The results of a test case are given in Fig. 7. It shows the $TM_{0(1)}$ dispersion curve for a sinusoidally corrugated waveguide, oriented in the z direction whose radius is described by

$$R = R_0(1 + \delta \cos 2\pi z/d) \tag{7}$$

where $R_0 = 1.5$ cm is the average radius, $\delta = 0.273$ is the normalized corrugation amplitude and $d = 1.67$ cm is the structure period. The dispersion relation shown by the solid line in Fig. 7 was derived point by point elsewhere [7] and tabulated in Table I (third column, $TM_{0(1)}$). The three special resonance frequencies ($f_0, f_\pi/2, f_\pi$) calculated by this code were used to generate the complete dispersion relation using the technique described in this work (6), and the results are again shown as a dashed line in Fig. 7. The two dispersion relations are in very good agreement, within 0.15%, over the entire wavenumber range $0 < \beta L < \pi$. Outside this range the dispersion relation is, of course, periodic.

As yet another check of the accuracy of our novel technique, we used the three specific resonance frequencies as calculated by a 2-D RF code [6] to generate the complete dispersion relation using (6). We then calculated additional points on the dispersion relation using the same numerical code and compared the results with the dispersion relation as calculated by three terms. Again, the results are in very good agreement ($>0.2\%$) over the entire wavenumber range ($0 < \beta L < \pi$).

In a similar way, we used three (out of seven) of the experimentally measured special resonant frequencies shown in Fig. 3 to generate a complete dispersion relation for actual periodic slow wave structures over the entire wavenumber range. These results are in excellent agreement with the experimentally obtained data ($>0.5\%$) for the remaining four resonances.

For even higher accuracy, more terms in (2) can be used. As was described in Section 2, the six period sinusoidally corrugated waveguide cavity was used to measure seven specific resonances for each transverse waveguide mode. We measured seven resonances associated with wavenumber values of $0, \pi/6, 2\pi/6, \cdots, 6\pi/6$ phase shift per period for each of the $TM_{0(1)}$, $TM_{0(2)}$ and $TM_{0(3)}$ modes. These are tabulated in Table I and plotted in Fig. 5. For this case (b) the complete dispersion relation (2) will be approximated for each mode by

$$f \cong \sum_{m=0}^{m=6} a_m \cos m\beta L \tag{8}$$

where the coefficients a_m, satisfying (9), are given by

$$\begin{aligned} a_0 &= \tfrac{1}{12}[f_0 + 2f_{\pi/6} + 2f_{\pi/3} + 2f_{\pi/2} + 2f_{2\pi/3} \\ &\quad + 2f_{5\pi/6} + f_\pi] \\ a_1 &= \tfrac{1}{6}[(f_0 + f_{\pi/3} - f_{2\pi/3} - f_\pi) + \sqrt{3}(f_{\pi/6} - f_{5\pi/6})] \\ a_2 &= \tfrac{1}{6}[(f_0 + f_{\pi/6} - f_{5\pi/6} + f_\pi) \\ &\quad - (f_{\pi/3} + f_{2\pi/3} + 2f_{\pi/2})] \\ a_3 &= \tfrac{1}{6}[(f_0 - f_\pi + 2f_{2\pi/3} - 2f_{\pi/3}] \end{aligned}$$

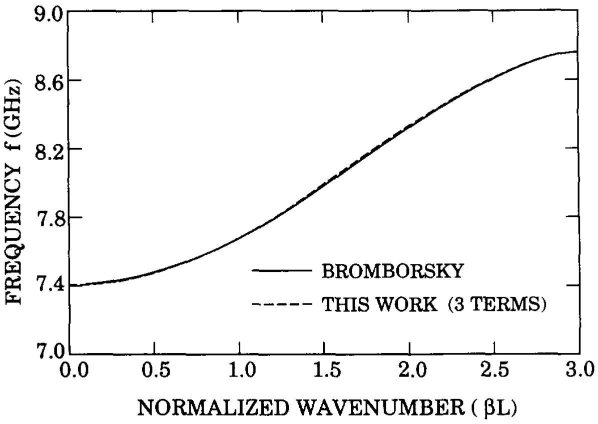


Fig. 7. Comparison of the $TM_{0(1)}$ dispersion curve of [7] (solid line) and this work (dotted line) for the sinusoidally corrugated waveguide (R_0 = 1.67 cm, δ = 0.273, d = 1.67 cm).

$$a_4 = \frac{1}{6}[(f_0 + f_\pi + 2f_{\pi/2}) - (f_{\pi/6} + f_{\pi/3} + f_{2\pi/3} + f_{5\pi/6})]$$

$$a_5 = \frac{1}{6}[(f_0 + f_{\pi/3} - f_{2\pi/3} - f_\pi) + \sqrt{3}(f_{5\pi/6} - f_{\pi/6})]$$

$$a_6 = \frac{1}{12}[f_0 - 2f_{\pi/6} + 2f_{\pi/3} - 2f_{\pi/2} + 2f_{2\pi/3} - 2f_{5\pi/6} + f_\pi]. \tag{9}$$

IV. Excitation of Pure Axisymmetric TM Modes in Periodic Slow Wave Circuits

The motivation for the design of the rod-wheel (Fig. 8(a)) and cage (Fig. 8(b)) launchers was outlined in Section II. There are considerable difficulties associated with the excitation of pure modes in a corrugated wall cavity. First, the presence of an input coupling probe often distorts the field pattern. Second, spurious modes can make interpretation of the experimental data difficult, especially in an overmoded system. Finally, the slow wave modes cannot be effectively excited by usual techniques. An improperly designed input coupler can launch just fast waves or will excite undesired TE waves and even asymmetric TM modes. It can also distort the desired field structure in the test circuit due to the inevitable existence of undesired field components induced by the probe. This might destroy the exact symmetry condition and adversely affect the measurement accuracy.

A suitable mode launcher should be able, together with a shorted slow wave circuit, to effectively excite (without distortion) and support both fast and slow waves. This is especially important for the lowest order passband modes where a large portion of the dispersion curve (in the region $0 < \beta L < \pi$) is dominated by slow waves. For an overmoded slow wave system the inverse is true (i.e., the fast wave portion of the dispersion curve is the largest). For effectively coupling to the slow wave space harmonics in the corrugated structure, the mode launcher should

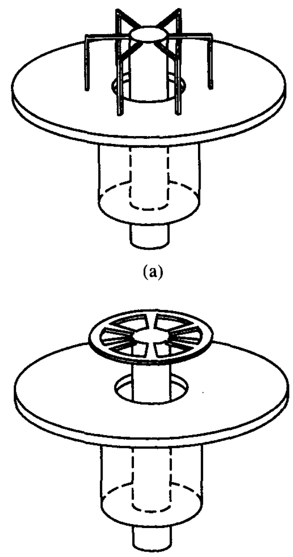


Fig. 8. Novel $TM_{0(n)}$ mode launchers used for excitation of the corrugated wall cavity: (a) cage antenna radiator, (b) rod wheel radiator.

be "appropriately" close to the corrugation. This is because the amplitudes of those space harmonics decay away from the corrugations. A single, on axis pin cannot provide efficient coupling and therefore cannot effectively excite slow waves around the "π" mode.

An ideal electric dipole has two important properties which may make it attractive for exciting corrugated wall cavities; 1) far from the dipole, the azimuthal magnetic field (H_ϕ) is minimum on axis, which is typical for slow waves, and 2) in the vicinity of the dipole, on a plane perpendicular to its axis, the only induced electric field component is E_z, which is compatible with the desired cavity field pattern in the absence of a launcher (Fig. 6). We will show that the two new antennas used in this work can be approximately treated as an ideal electric dipole and therefore may serve as a suitable source for driving the corrugated wall cavity. Both antennas yielded excellent experimental results.

The cage antenna coupler is a complex current loop formed by parallel connection of many single loops symmetrically arranged around a center conductor, as shown in Fig. 8(b). It is characterized by two features. First, its axial symmetry will ensure excitation of axially symmetric fields. Second, it will only excite the azimuthal component of magnetic field H_ϕ and therefore is equivalent to an ideal electric dipole radiator in the z direction located at the origin of a set of spherical coordinates (Fig. 9). The axial component of the magnetic field (H_z) produced by the radial currents flowing on the cage top will be cancelled because of axial symmetry. The electric and magnetic field components excited by the cage radiator in the

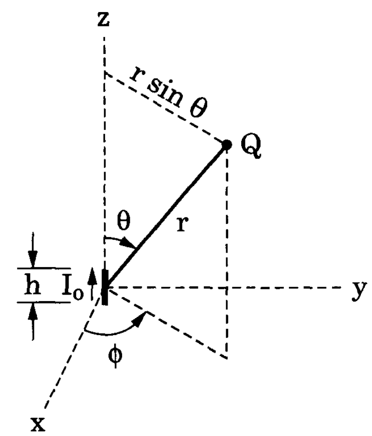


Fig. 9. Schematic diagram of an ideal electric dipole radiator together with the notation used in our work.

free space may therefore be approximated by a short (much less than a wavelength), ideal electric dipole in the z direction [13]. Considering the near field at $\theta = 90°$ (mirror plane position) we have

$$|H_\phi| = \frac{I_0 h}{4\pi r^2}$$

$$|E_r| = 0$$

$$|E_\theta| \Rightarrow \frac{I_0 h}{4\pi r^3}\frac{\omega\mu}{\beta^2} \tag{10}$$

where $\beta = \omega\sqrt{\mu\epsilon} = 2\pi/\lambda$, h is the length of the antenna, and I_0 is the total current flowing through the antenna.

Equation (10) indicates that on the surface of a plane going through the origin and perpendicular to the radiator, the excited E field is parallel to the z axis. This is consistent with the desired field structure (Fig. 6). The introduction of the coupling antenna is therefore not expected to bring about a substantial disturbance in the field of the corrugated wall cavity.

At a distance far from the antenna, the only important terms of E_r and H_ϕ are those varying as $1/r$. The Poynting vector is then completely in the radial direction, and the time-average power flow is

$$P = \tfrac{1}{2}\,\mathrm{Re}\,(E_\theta H_\phi^*) \propto \sin^2\theta \tag{11}$$

Equation (11) shows us another very interesting characteristic of the cage antenna, i.e., it will favorably excite the slow wave modes in the corrugated waveguide cavity. This is because only H_ϕ and E_θ can be excited. E_θ is equivalent to the vector sum of E_z and E_r in a cylindrical coordinate system and has no significant value in the region near z axis ($\theta = 0$) and far from the radiator.

Another type of coupling device used to excite the test model is the rod-wheel antenna as depicted in Fig. 8(b). It is also equivalent to an electric dipole radiator because the displacement current ($I_D = \partial\epsilon\overline{E}/\partial t$) between the wheel and the mirror ground plane has only one component in z direction and forms a rectangular closing loop for the antenna current (equivalent to the current flowing in the axial conducting wires around the center in the case of the cage antenna structure). Thus the analysis for cage antenna is approximately valid for the case of the rod-wheel structure. This structure proved to be superior to a conventional E probe coupler which cannot effectively excite slow wave modes and will inevitably distort the field structure near the mirror plane (leading to poor accuracy in the measurements of the specific resonant frequencies).

V. Discussion and Summary

It is usually difficult to completely, accurately, and unambiguously measure the dispersive characteristics of specially periodic structures, especially of the higher passband modes [14].

The technique presented here is simple to use. Furthermore, it is general in nature and only the mode launcher is structure/mode specific. The analytic interpolation between points on the dispersion relation takes into account the intrinsic characteristics of the slow wave circuit. This technique is unique in that it has been successfully applied to the fundamental as well as higher order passband $TM_{0(n)}$ modes.

We have shown that to a high degree of accuracy, the complete dispersion relation of lower passband modes can be expressed in a simple form (6) with only three coefficients, A, B, and C. In this case, (6) enables one to directly calculate the phase and group velocities at any point along the dispersion relation. The expressions derived for the velocities are given in (12) and (13):

$$v_{\text{phase}} = \frac{[2L\pi f_{\pi/2}\,(1 + \Delta f/f_{\pi/2})]}{\cos^{-1}\,([(\beta^2 - 4C\Delta f)^{1/2} - B]/2C)}, \tag{12}$$

$$v_{\text{group}} = \partial\omega/\partial\beta = 2\pi L(B\sin\beta L + 2C\cos\beta L\sin\beta L), \tag{13}$$

where $\Delta f = f - f_{\pi/2}$.

Since the interpolation between points on the dispersion relation is based on the intrinsic characteristics of slow wave circuits (rather than an arbitrary lumped element equivalent circuit) the accuracy depends on the number of terms used in the expansion of (2). With seven terms it can be accurate to better than one-tenth of one percent even for higher passband modes. For the case of three terms the technique requires a knowledge of three special resonance frequencies which can be determined experimentally or numerically. With this in hand, complete dispersion relations were constructed, and additional resonances were compared with constructed dispersion curves. This was done for three passband modes ($TM_{0(1)}$, $TM_{0(2)}$, $TM_{0(3)}$) with the techniques available for acquiring the special resonance frequencies, and the results are tabulated in Table II.

For the experimental studies reported in this work, we developed two novel mode launchers suitable for exciting $TM_{0(n)}$ passband modes in a rippled wall waveguide: a cage antenna radiator and a rod-wheel antenna. Both

TABLE II
THE ACCURACY OF THE SYNTHETIC TECHNIQUE FOR THE CALCULATION OF THE COMPLETE DISPERSION RELATION USING THREE TERMS (Eq. (6))

Passband Mode	Method for acquiring the special resonance absorption frequencies			
	Experimental (this work)	2-D RF Code Superfish[6]	Bromborsky & Ruth[7]	Lou & Carmel[9]
$TM_{0(1)}$	<0.510%	<0.210%	<0.500%	<0.085%
$TM_{0(2)}$	<0.034%	N/A	<0.130%	<0.096%
$TM_{0(3)}$	<1.300%	N/A	<0.6405%	<0.160%

launchers proved to be capable of exciting the desired space harmonics with little distortion of the field structure in the corrugated wall cavity. With these mode launchers, the cavity was free from spurious modes and it was possible to excite the π mode (see Figs. 3 and 5), which is normally very difficult experimentally using other launchers.

From a practical point of view, the most relevant comparison is probably between the experimentally obtained frequencies and those calculated by Superfish (which is widely accepted as a quasi-standard); for this comparison, see Table I. The two agree to better than 1%. This difference of 1% is probably due to the mechanical tolerances of the corrugated wall structure and cannot be used as an estimate for the accuracy of the experimental technique.

We also tried, successfully, to excite the $TM_{0(4)}$ passband mode in a rippled wall cavity and indeed uncovered seven resonances, as expected. Interpretation, however, is difficult because the dispersion relation, $f = f(\beta)$, is not a monotonic function of β in the interval $0 < \beta L < \pi$. Even though the resonances are equally spaced in β, it is difficult to match specific resonance to a specific wavenumber for a non-monotonic dispersion relation. The information extracted from the $TM_{0(4)}$ measurement is limited for the time being to the upper and lower cutoff frequencies for that mode. Agreement in this respect is also better than 1%. The technique for determination of the complete dispersion relation of very high order $TM_{0(n)}$ modes when $n \geq 4$ is now under further development.

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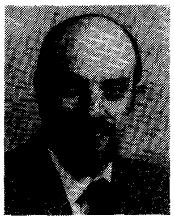

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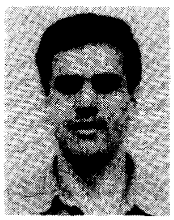

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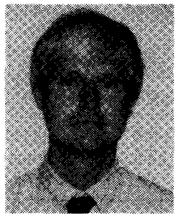
A. Bromborsky, photograph and biography not available at the time of publication.



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