Analysis of a Dynamics-based Algorithmic Compression Process for Audio Waveforms

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Abstract

The goal of this research is to use concepts based on chaotic dynamics to match and replace arbitrary audio waveforms with chaotic oscillations. Using these concepts provides compression by replacing audio waveforms with minimum critical points. This technique is being developed for commercialization by Syncrodyne Systems Corporation and researched at the Center for Advanced Technology Development at the Rochester Institute of Technology. The fundamental assertion that we want to show is that by maximizing **three measures** of chaotic behavior we will maximize the probability of successfully matching arbitrary audio waveforms. These measures are the **Lyapunov exponent**, the **metric entropy**, and the **spectral bandwidth**.

Definition:

- Compression of digital waveforms has been studied and implemented in various ways for years. We look to show N:1 compression by replacing 16-bit, 44.1 ksps (CD quality) digital audio waveforms with oscillations from a chaotic system that is a good representation of its shape.

- The chaotic oscillation can be regenerated because it is produced by a mathematical operation. For example if 80 sample points are replaced by 4 numbers, called *d-bites*, then the compression ratio is: 80:4 or 20:1

- We assert that if we optimize three measures of chaotic behavior, we increase the probability of matching arbitrary digital waveforms, thus maximizing signal quality.

What is Chaos?

- What is chaos? After over one hundred years of research, study, scrutiny, analysis, modeling, and experimentation, this is a question that still eludes a definitive answer.

- Chaos is better defined by its behavior or its properties. Chaos is a descriptive term for the properties of a system in motion, or a *dynamical system*. Therefore chaotic behavior is often referred to as *chaotic dynamics*. More specifically, chaotic dynamics is a type of behavior that occurs in nonlinear systems.

- Chaos has been observed in many types of systems. It is especially common in electrical and electronic circuits. We will explore a specific electronic circuit called the Colpitts Oscillator.

Chaos – Colpitts Oscillator

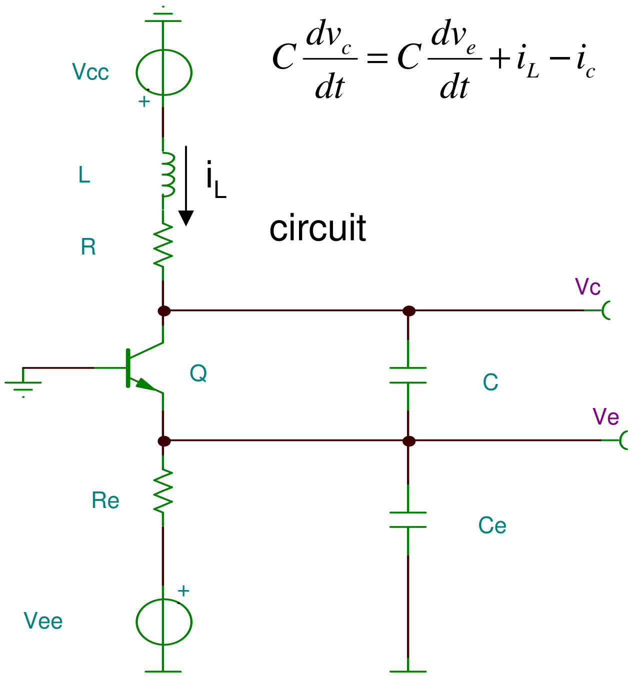
circuit equations

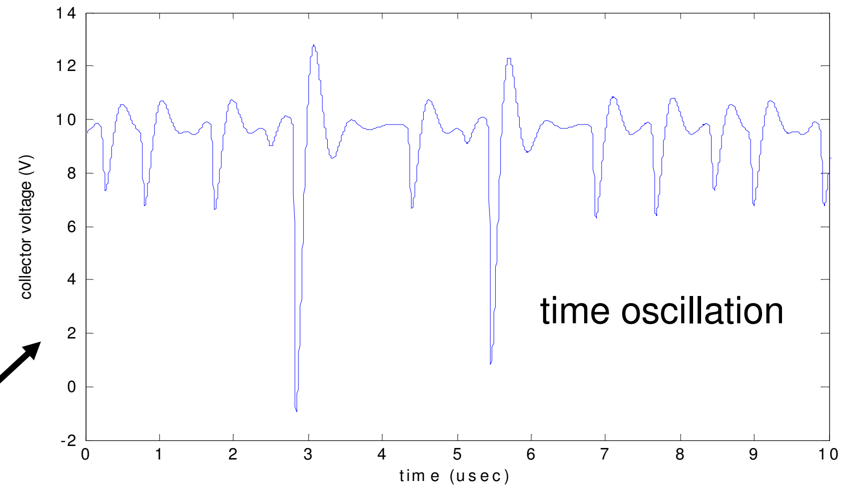
$$L\frac{di_L}{dt} = V_{CC} - v_c - (R + R_L)i_L$$

$$C_e\frac{dv_e}{dt} = i_L - \frac{v_e - V_{EE}}{R_e}$$

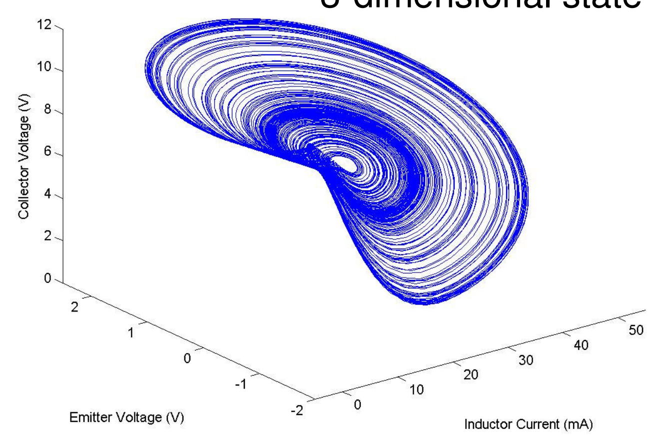
$$C\frac{dv_c}{dt} = C\frac{dv_e}{dt} + i_L - i_c$$

circuit





3-dimensional state-space

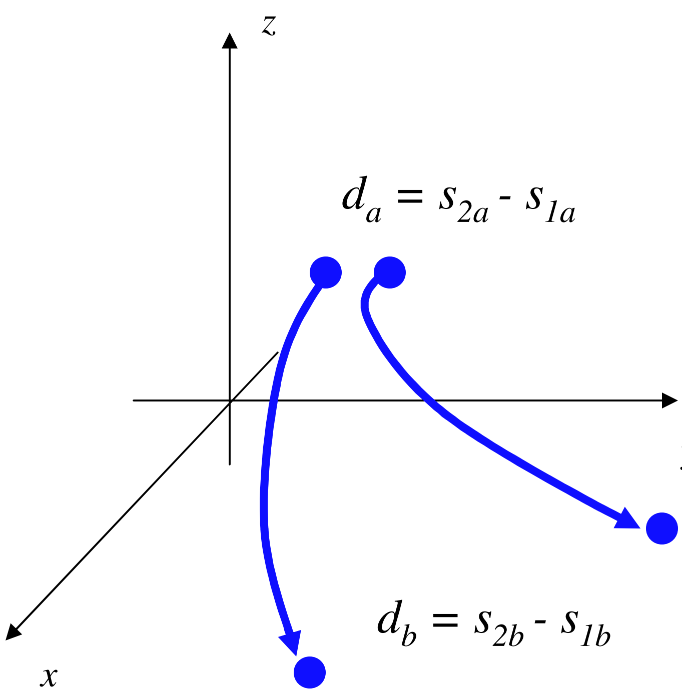


Definitions of Measures

The three measures of chaotic behavior that we will seek to optimize are:

- Lyapunov exponent – a measure of the divergence and convergence of chaotic trajectories.
- Metric entropy – provides a measure of the diversity of the waveforms produced by the chaotic oscillator.
- Spectral bandwidth – chaotic oscillations are naturally broad-band thus providing a rich pool of potential signal shapes.

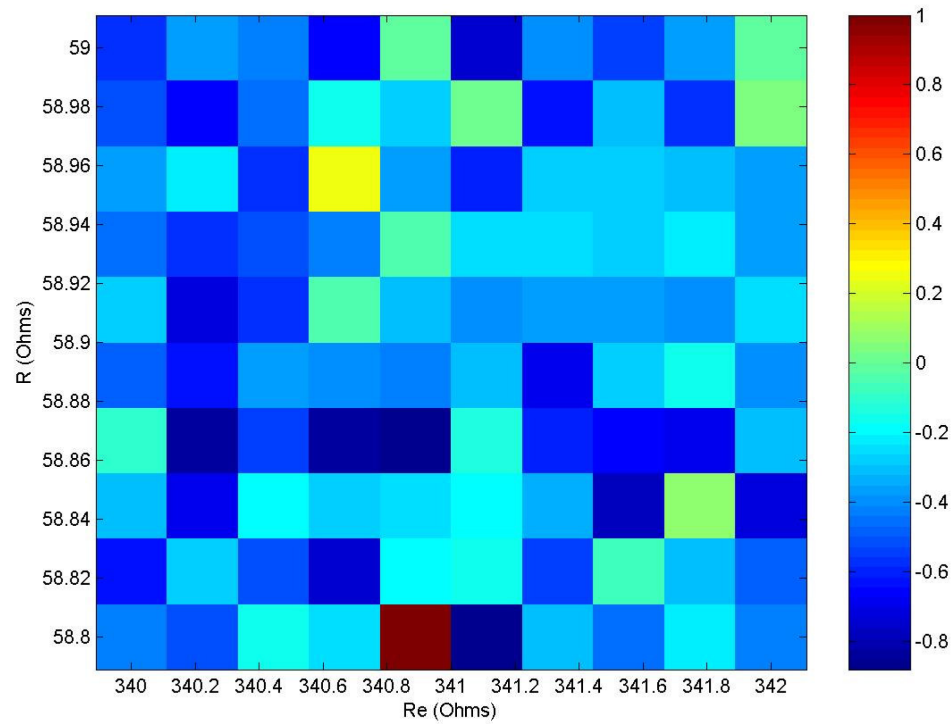
Lyapunov Exponent



$$d_b = d_a e^{\lambda \Delta t}$$

λ– lyapunov exponent

+ (chaotic)

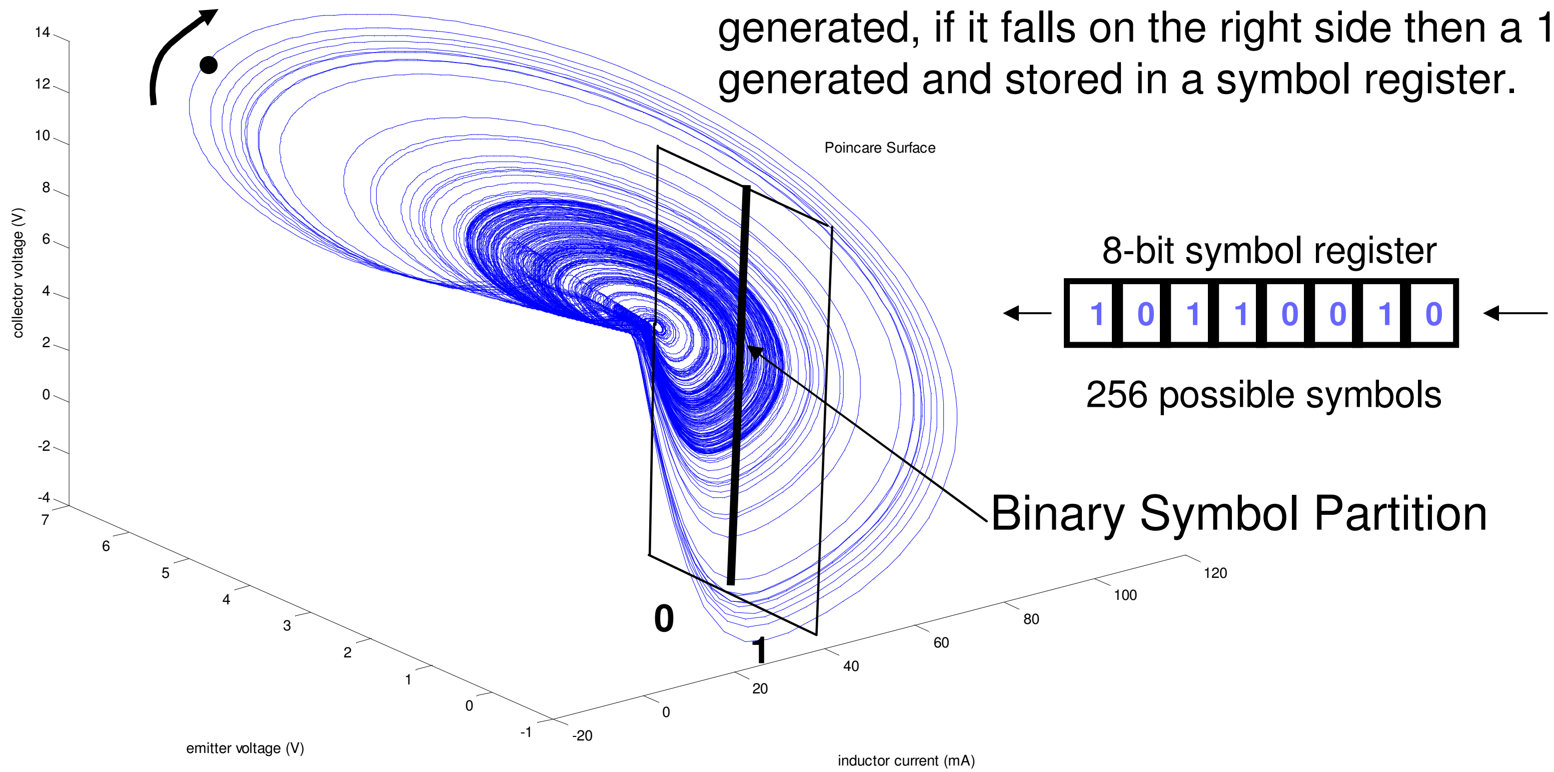


As we vary the circuit parameters, R and Re, we look at the average lyapunov exponent for the system to see where it is maximal and positive.

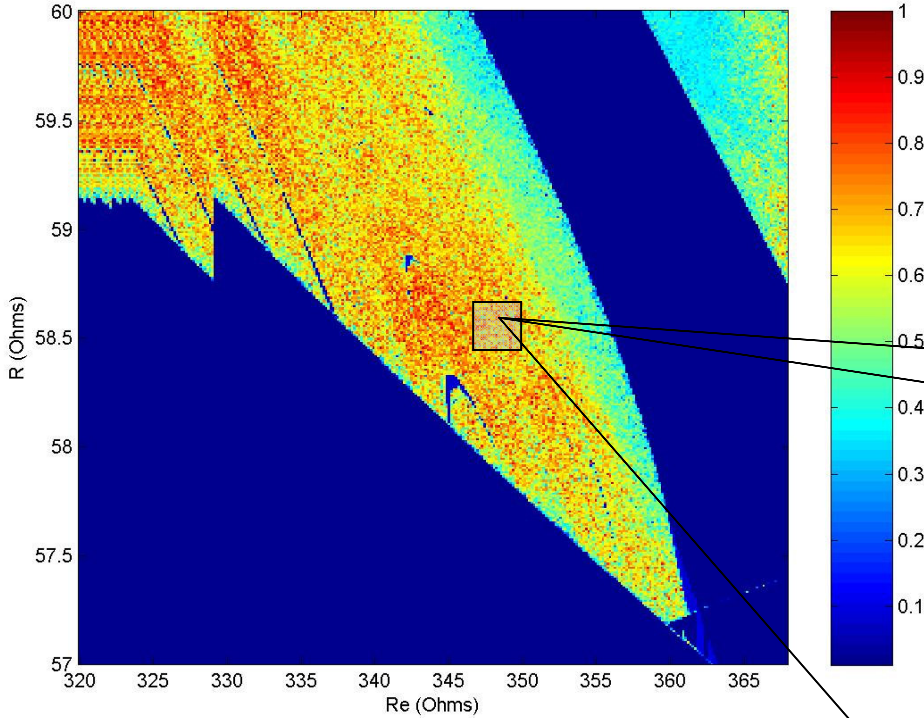
Metric Entropy

•Every time a trajectory passes through the surface it leaves a point.

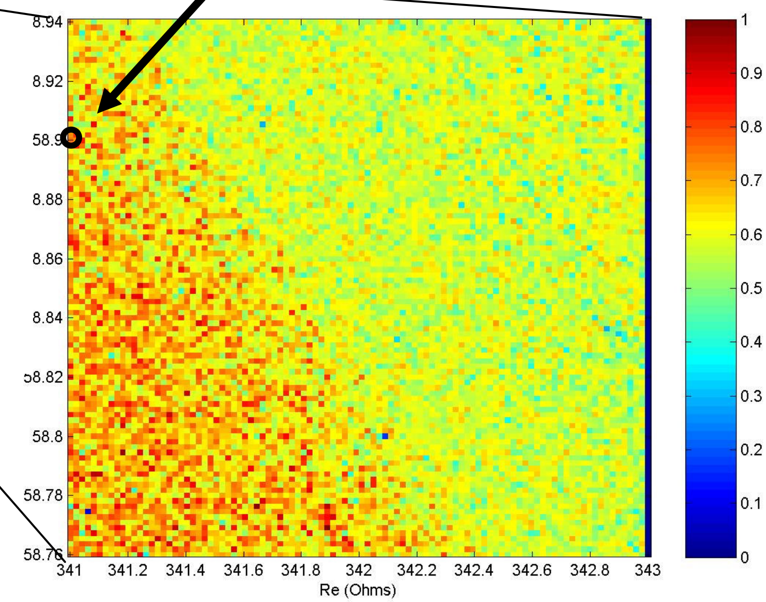
•If the point falls on the left side a 0 is generated, if it falls on the right side then a 1 is generated and stored in a symbol register.



Metric Entropy

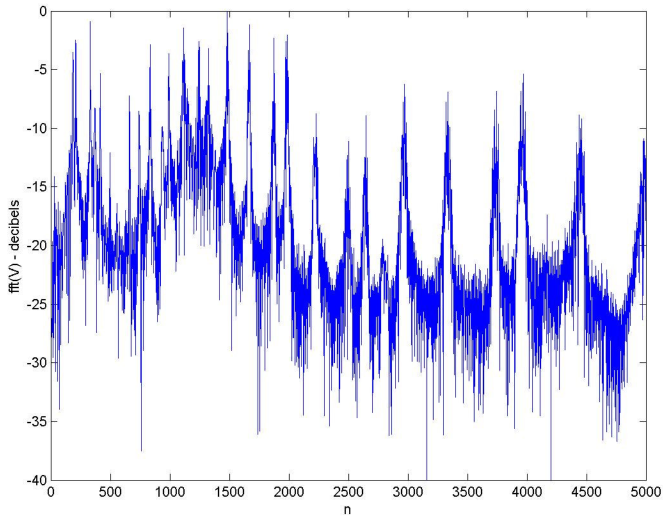


- This analysis generates a *fractal* image that tells us the best values for Re and R.
- In this example we used Re = 341 Ω and R = 58.9 Ω.

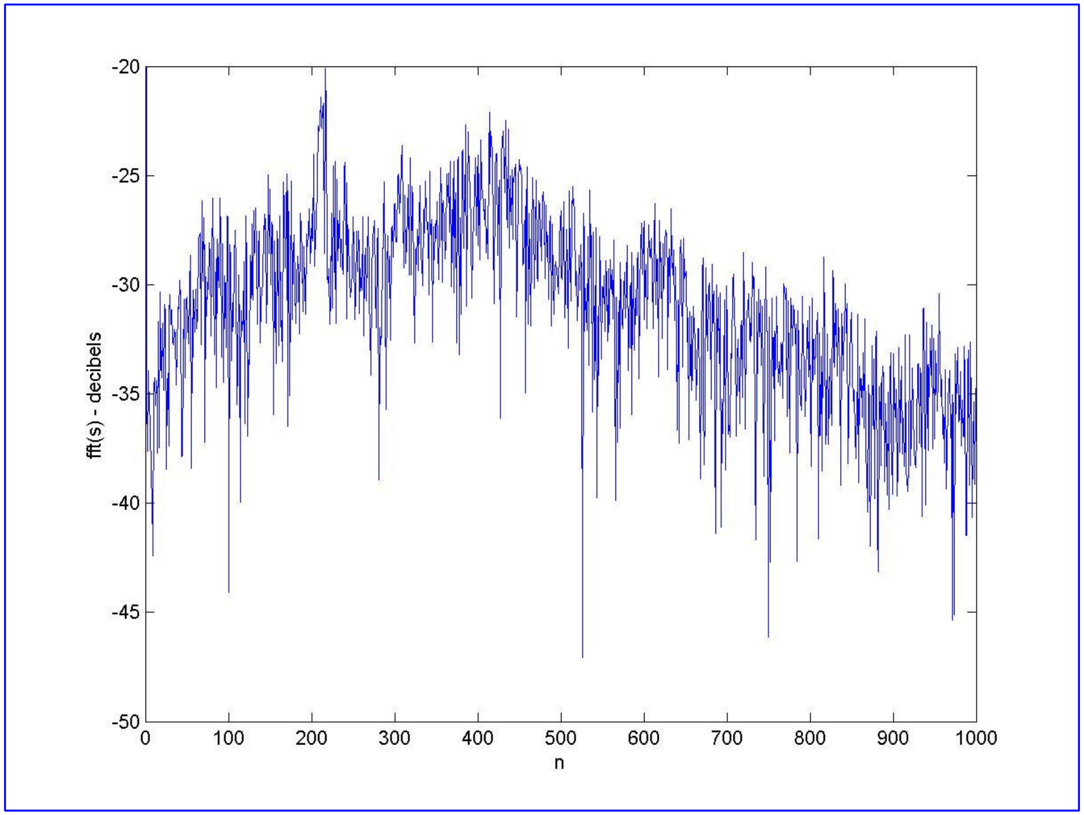


Frequency Content

FFT for our test sound waveform



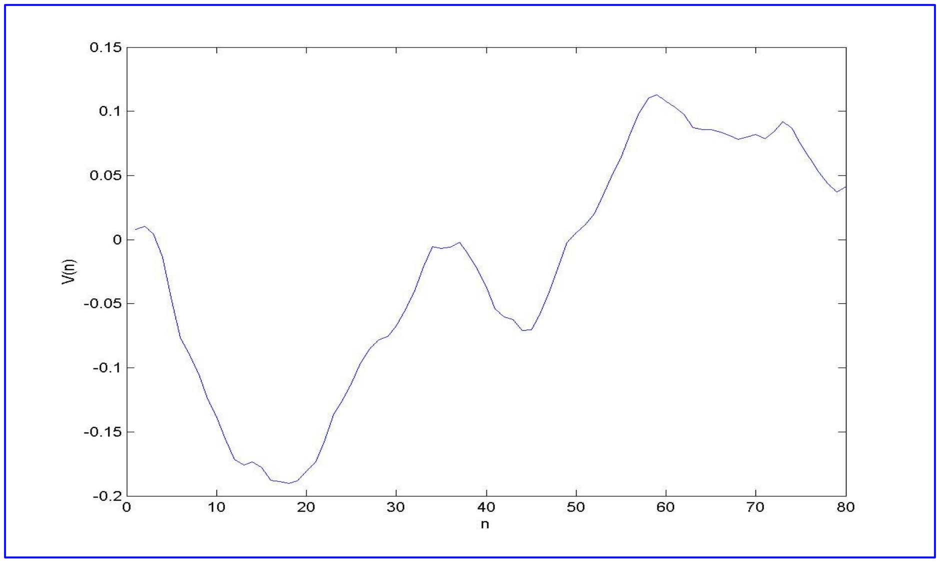
FFT for Colpitt's oscillation



- We seek to broaden the frequency spectrum
- The broader the frequency spectrum, the higher the probability of fitting the original sound file.

Approach

Replace a long piece of a sound file with points that reconstruct a matching waveform



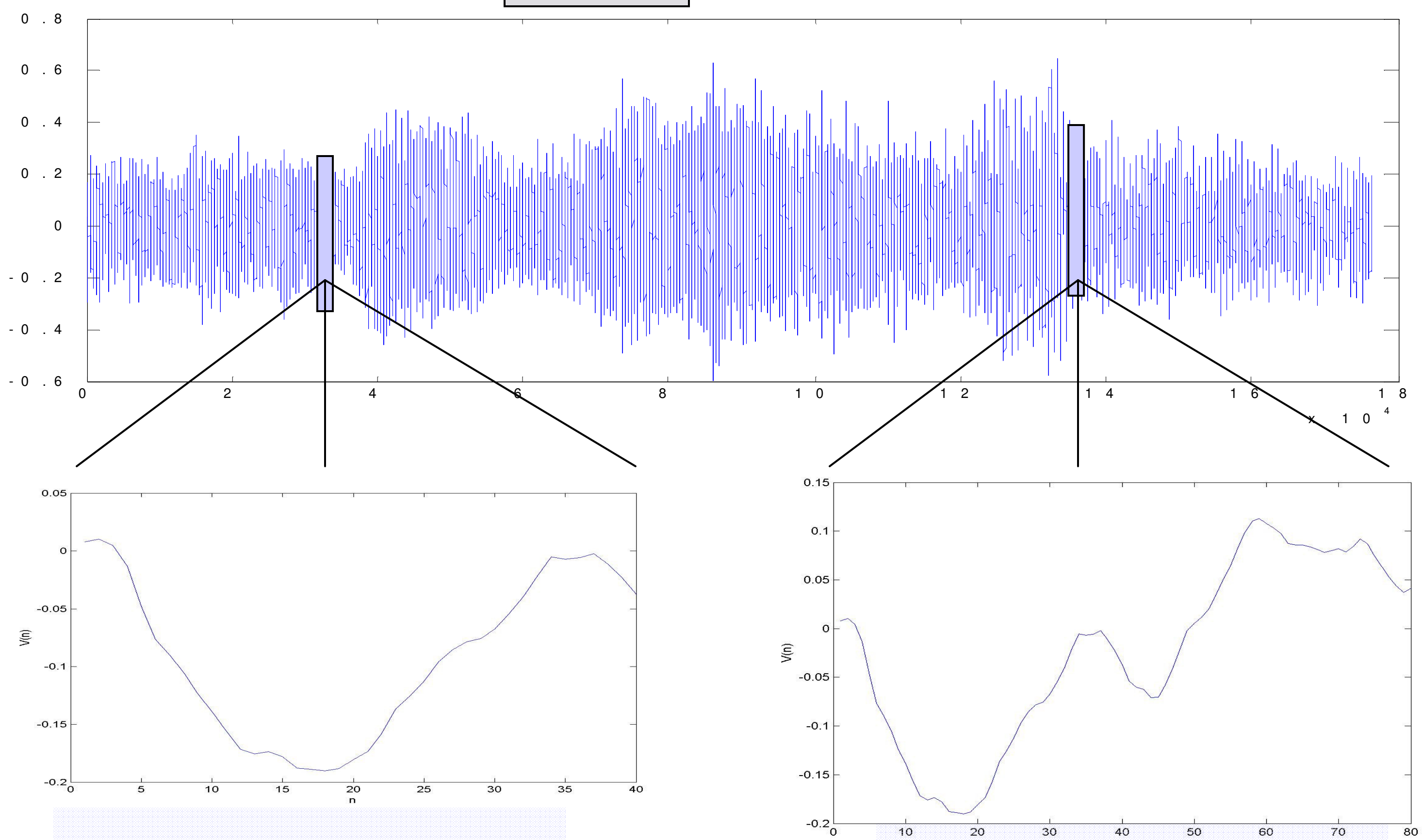
D-bites (16-bit)
D1
D2
D3
D4

80 points of a sound wave replaced with 4 critical points, resulting in a 20:1 compression ratio

Algorithm Application - Example

Sample Sound File

Song Piece

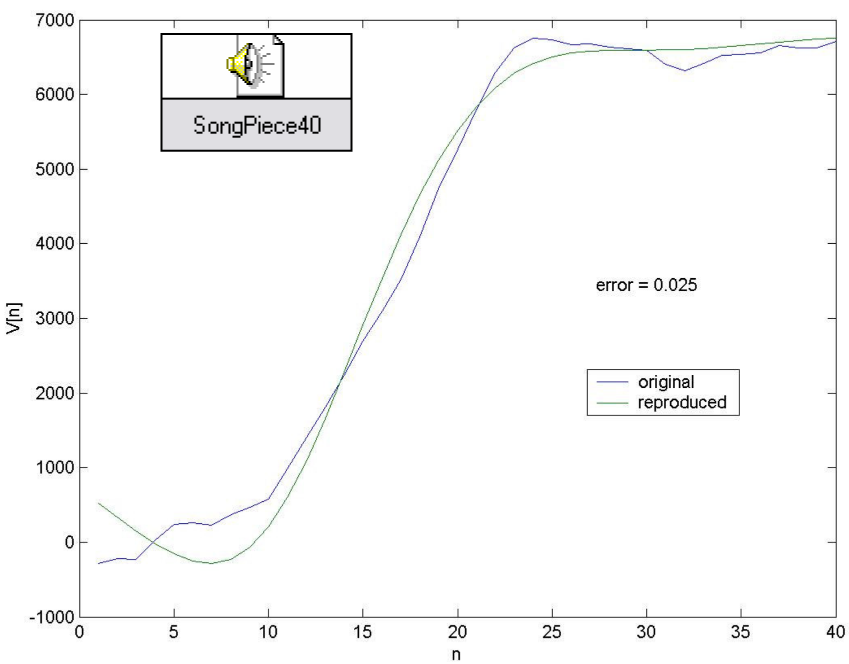


40 points – 10:1

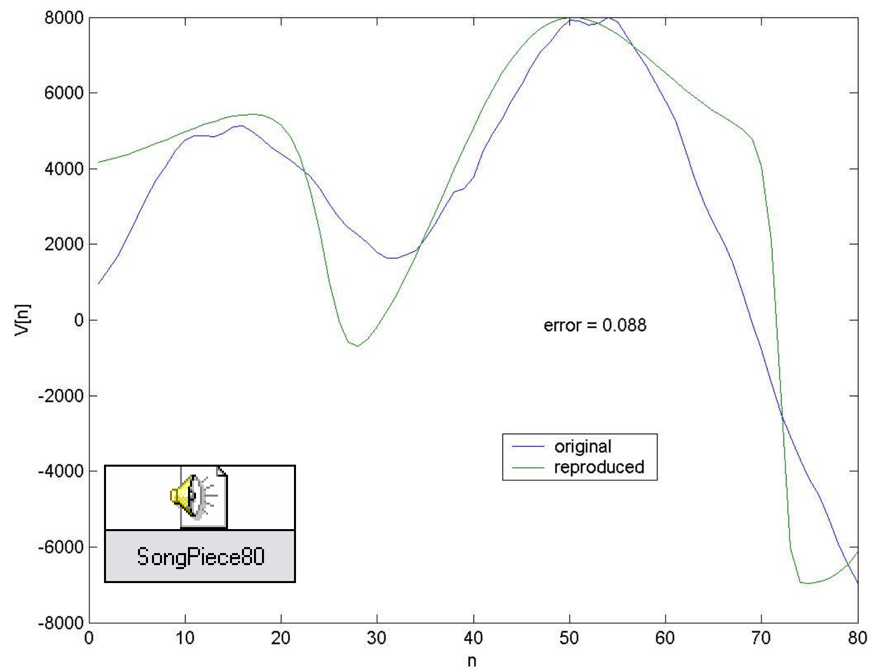
80 points – 20:1

Algorithm Application - Example

Compression Ratios



- 10:1 Compression
- Error Analysis



- 20:1 Compression
- Error Analysis

Conclusion

- Summary of Results
  - We have shown that CD quality sound files can be compressed using a dynamics-based approach.
  - We have established our premise that the optimization of measures of chaotic dynamics maximized the probability of matching arbitrary sound file pieces.
  - We are developing methods to improve the algorithm for sound quality, compression ratio, and processing speed.