A MATHEMATICAL ANALYSIS OF THE SPECTRAL DENSITIES OF SUPER-GAUSSIANS.

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The rectangle function, $f_\infty(x) = \begin{cases} 1 & x \in (0,1) \\ 0 & x \notin (0,1) \end{cases}$ and the Gaussian, $f_2(x) = e^{-x^2}$ are central to the mathematical theory of image representation. Their Fourier transforms — or spectral densities — are, respectively, the SINC function and another Gaussian. There is a sequence of functions, the super-Gaussians, $f_N(x) = e^{-x^N}$, that make the transition from the Gaussian to the rectangle function in that the second term in this sequence *is* the Gaussian, and $\lim_{N\to\infty} f_N(x) = f_\infty(x)$. We have investigated the properties of the spectral densities of these functions,

$$\phi_N(\xi) = \int_0^\infty e^{-x^N} \cos(\xi x)dx$$

in order to understand how they make the transition from the Gaussian to the SINC function, with an eye toward using the super-Gaussians in image interpolation.

In this talk, we present out preliminary results, which show that the spectral densities exhibit complicated and interesting behavior as they approach the SINC function. We show that for even values of N, $\phi_N(\xi)$ has an infinitude of roots, whereas for odd values of N, $\phi_N(\xi)$ has a finite number of roots. We discuss a method that combines theoretical analysis and high-precision computations to investigate the actual number and location of roots of $\phi_N(\xi)$ for odd N. We show how we have used this method to generate hypotheses about the number of roots, and we conclude with a discussion of our plans for further research.